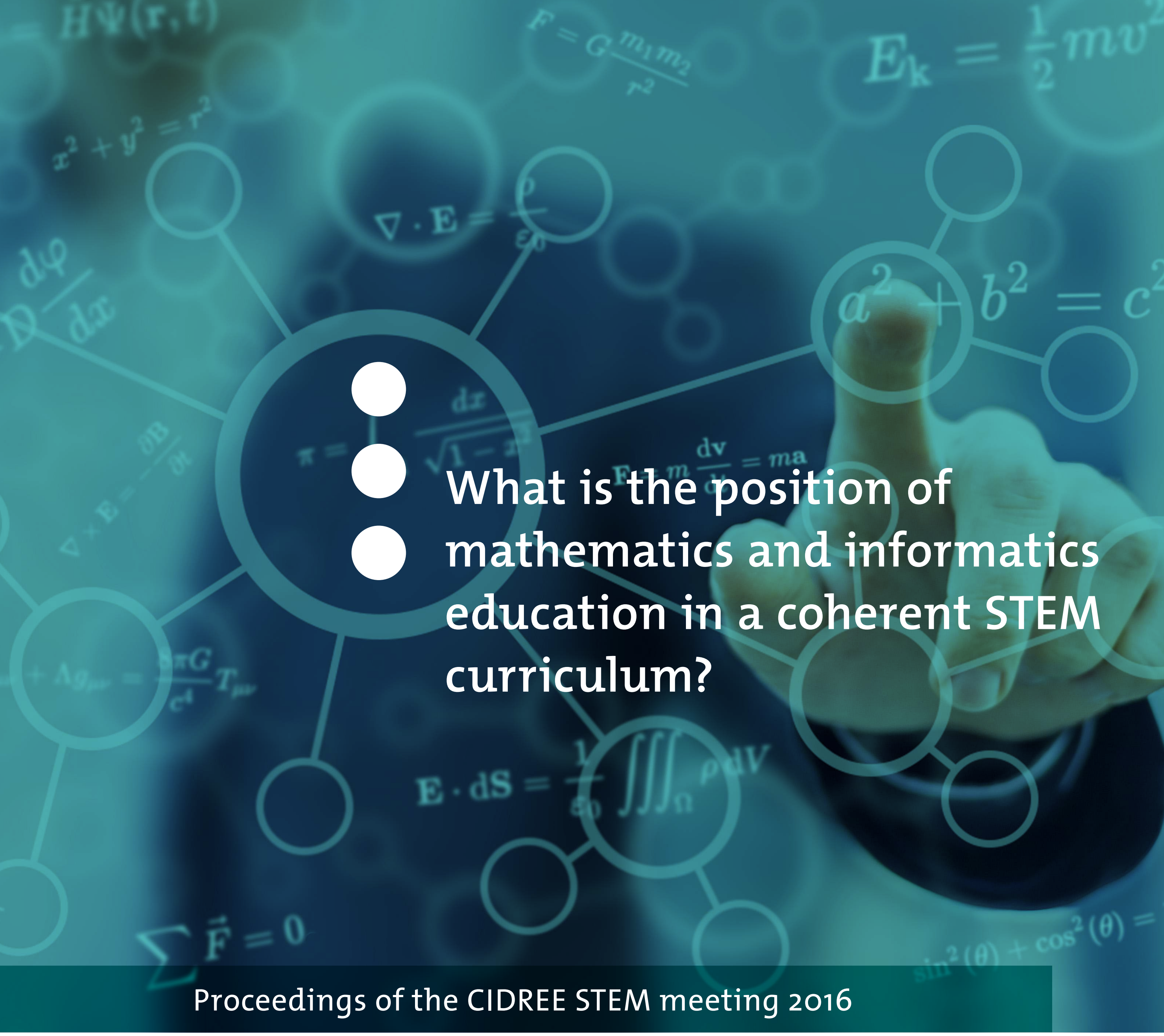

What is the position of mathematics and informatics education in a coherent STEM curriculum?

Proceedings of the CIDREE STEM meeting 2016

slo

CIDREE

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CIDREE
Consortium of Institutions for Development and Research in Education in Europe

slo
Netherlands institute for curriculum development

Colophon

CIDREE is a network of educational organisations involved in curriculum development and/or educational research, set up in 1990 to establish closer working relationships at a European level.

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Contents

Computer Based Mathematics: a global progress report

Adrian Smith & Alec Titterton

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Abstract

Five years ago Wolfram launched CBM as a 25 year ambitious mission – to persuade countries to reject traditional mathematics curricula and replace them with a fundamentally different curriculum that is computer based and problem centric. The journey is tortuous but we now have activities at various stages of progress in a handful of countries with one, Estonia on the cusp of a national roll- out.

During this CIDREE event you will hear from colleagues in Estonia and Ireland (and possibly Sweden?) about the practical experiences and challenges of implementing CBM. These will address both the cultural and mind-set shifts that are needed even before tackling the infrastructure and stakeholder challenges that are at the heart of a successful deployment.

As providers of the concepts and technologies that sit behind these implementations, our objectives today are slightly different. We want to explore how our ideas have evolved over the past five years, where there have been incremental or step-changes and how the landscape has shifted around us. We want to ask and seek to answer FIVE key questions:

- Is the overarching vision and concept of computer-based mathematics as relevant and critical today as it was 5 year ago?
- How do our ambitions for mathematics translate into the broader STEM agenda?
- Is the emerging focus on ‘computational thinking’ aligned with or at odds with our ambitions and thinking?
- What progress have we made on defining a set of outcomes that underpin and drive a computer based mathematics curriculum?
- Where is assessment in all this?

And of course in answering these questions we will touch on where we head next, and why.

The incresing relevance of Computer Based Mathematics

During the past five years the landscape has changed and with it our vision and ambition. What started out as a solely mathematics initiative has now extended across the STEM agenda and equally importantly has embraced the emerging focus on computational thinking. This paper definitely falls into the ‘unfinished’ category as there is at least another twenty years to go. But the lessons we have learned are already worth sharing from our journey so far and to help shape where this important initiative heads next.

From a mathematics perspective the issues haven’t fundamentally shifted. Around the world we still have children sitting in classrooms bored and bemused by the process-based, hand-calculation subject that is traditional, educational mathematics. We still have Universities and Industry telling us that students that come out of schools don’t have the skills and problem solving capabilities to equip them for life, work or further education. We still have economies desperate for better quantitative skills to help them compete in an ever more demanding global environment.

And there is another level of challenge too. Whether it is managing one's own health or the health of the planet as a whole we need a new generation of quantitative problem solvers that understand and embrace these challenges in a constructive fashion. As the world heads towards a place where economic growth simply isn't sustainable long-term, the ability to 'do' and understand the simple maths around such problems is genuinely critical.

From all of these perspectives computer based mathematics remains as relevant now as it was five years ago. What also remains are the same challenges of convincing governments, policy makers, educational institutions and teachers to make the shift. There are plenty who see the benefits but some of the barriers can be particularly obstructive.

Chief amongst these is the adherence to 'point system' approaches to university entrance based on assessments which lend themselves to drill and practice learning. These drive behaviours in both children and parents to such an extent that the introduction of problem-solving curricula is exceptionally challenging. Sometimes, as in Estonia and Ireland, there is a government, policy level vision but in the absence of that more forward-thinking University

The Broader STEM agenda

One of the mistakes we confess we probably made was in the branding of the initiative and the narrowness of the focus on mathematics!

It has become increasingly obvious to us that computer-based, problem-centric approaches are key to all STEM subjects and potentially beyond. Why is this? It's because engaging and motivating students requires realistic, interesting problems and, across the STEM curriculum, these can't be solved by hand.

As we have developed CBM content the problems we have selected have been drawn from across the STEM curriculum. We have built our CBM materials around topics as diverse as how to win a bicycle race, marketing the 'best' mobile phone, controlling a quadcopter or deciding whether boys are better than girls at mathematics! Our aim has been to choose problems which will:

- Motivate students to enjoy mathematics and want to learn more
- Build mathematical skills by introducing increasingly complex concepts rather than increasingly complex processes and procedures
- Build an understanding of and competence in using an iterative four step problem solving methodology that has broad applicability.
- Develop complementary coding skills (see section 3)
- Address a rather different set of mathematics outcomes than has been seen in traditional mathematics education (see section 4)

All of these skills and competences that we saw as at the heart of computer based mathematics are at the heart of computer based STEM. Moreover, it could well be that it is easier to introduce computer-based methodologies into other STEM studies than it is into mathematics where there remains often irrational resistance to moving away from hand-calculation.

Computational Thinking

"Computational thinking is a fundamental skill for everyone, not just for computer scientists. To reading, writing, and arithmetic, we should add computational thinking to every child's analytical ability. Just as the printing press facilitated the spread of the three Rs, what is appropriately incestuous about this vision is that computing and computers facilitate the spread of computational thinking."

Jeannette M. Wing - Professor of Computer Science and head of the Computer Science Department at Carnegie Mellon University, and Corporate Vice president, Microsoft Research

From the very outset we integrated coding into our computer-based mathematics curricula. It was always seen as a natural part of the process – translate a real, complex problem into a mathematical model and then translate the model into code to execute the solution.

Of course, as we will explain shortly, one of the outcomes we want to achieve from computer-based mathematics is the ability to select already available techniques and tools which students can apply to problems. We aren't advocating that they code every solution from scratch. But coding builds understanding in a unique way. I always think of the analogy of how preparing to teach a subject to someone else is the greatest driver of building full understanding. In the same way writing code (teaching the computer?) is a unique way of fully understanding concepts and the underlying mathematics. If you aren't convinced of this get a set of students to write a few lines of code to plot the imagination inspiring Mandelbrot Set. Writing the code gets a level of conceptual understanding that simply isn't possible otherwise.

So we were thinking this way before the term 'Computational Thinking' became as fashionable as it now is. But now it's in vogue it has certainly helped crystallise some of the ideas.

Jeanette Wing, quoted above, isn't the only clear thinker on the subject but I will just stick with her because I think she has clarified some of the key themes very clearly. I have paraphrased some of these themes:

- It's more than coding – it's thinking like a computer scientist, conceptualising and applying multiple levels of abstraction
- It's about fundamental note rote skills – it's something every human needs to know but isn't a mechanical process.
- It's a way that humans, not computers, think – we are not trying to get humans to be dull and boring like computers but rather to unleash their imagination and creativity
- It complements and combines mathematical and engineering thinking – the constraints of the underlying computing devices force us to think computationally, not just mathematically. Being free to build virtual worlds enables us to engineer systems beyond the physical world.
- It's about ideas, not artefacts - the computational concepts we use to approach and solve problems, manage our daily lives, and communicate and interact with other people are more important than the hardware and software we produce
- It's for everyone, everywhere – at some point it will become a natural and normal part of every aspect of human activity.

I think these themes encapsulate what we have been trying to do with computer-based mathematics and crystallise some other dimensions too. As we develop and grow computer-based STEM it needs to integrate computational thinking so that it becomes a stepping stone to delivering the final point that Jeanette makes – everyone sees it as a natural part of their activity and endeavours.

Smith, A. & Titterton, A. (2016) *Computer Based Mathematics: A global progress report*. [Working paper submitted for CIDREE-STEM 2016, December 22nd].

Computer-Based Mathematics/STEM Outcomes

One of the key challenges we have faced is defining, clearly and explicitly, the educational outcomes that drive and complement our content.

In the mathematics domain we have been very clear that we are building a replacement curriculum and as such the traditional learning outcomes are as in need of replacement as much as the content.

At the outset we made comparisons of the sorts of topics we expected CBM to cover in comparison to traditional curricula, for example:

Traditional Mechanistic topics	Computer-based mathematics Problem-centric
• Complete the square • Invert a matrix • Simplify a surd or recurring decimal • Solve simultaneous equations • Calculate angles in a triangle • Use the chain rule	• Should I insure my laptop? • How do I design controls for my game? • How can I make a perfect login password? • What makes a beautiful shape? • How far can I compress photos, video or music before I notice? • How many words do I know?

These problems were then mapped to an initial set of outcomes. We have now taken a step forward relaunched the CBM outcomes. Now, not only have we mapped the required skills of problem- solving as identifiable outcomes within our problem-solving cycle, we have begun to map out the primary contexts and areas of maths that are vital for computer-based mathematics students of the future to learn.

All new CBM materials are tethered more explicitly around the problem-solving cycle with teacher modalities and student material clearly showing which part of the problem-solving cycle the student is currently operating within. These materials can be deployed at one of three levels:

- Beginner—Directed learning. CBM students at any age who are unfamiliar with the problem- solving cycle and unfamiliar with the tools needed to operate successfully in a computer- based environment. For these students the directed teaching and learning deployment is constructed with full teacher instructions, student guidance, formative assessment opportunities, interactives, example datasets, tasks and review points.

- Intermediate—Guided learning. CBM students who are more experienced in problem solving but still need some guidance about the steps to take or the tools to use. For these students the guided learning deployment consists of a suitable scaffold for the problem solution with hints at questions to ask, tools to use, and places to find information.

- Advanced—Independent learning. CBM students who are experienced problem solvers. Self- starters who know how to problem solve, know what tools are available and where to find the necessary information to enable progress to be made. This deployment consists of topical problems to pose, with teacher guidance only for areas that the students may investigate and example solutions to the problems.

To see a full list of the required outcomes from a mainstream CBM curriculum, please visit http://www.computerbasedmath.org/materials/math-education-outcomes.php#detailed-view-dq and download the pdf.

Assessment for Computer Based Mathematics/STEM

The next obvious challenge is how to assess these outcomes in robust and engaging ways.

As CBM is moving away from focussing the assessment on the ability to predominantly hand-calculate a numeric answer to a numeric question, the options for assessment methods are broader. There is an easy option to tick the boxes of being able to use a list of tools to compute answers in a
similar way to the traditional curriculum, but the CBM curriculum is much more about the adaptable problem-solving abilities of students rather than their repetitive mechanistic abilities.

We are currently working on providing assessment guidance on a number of levels. We are writing exemplar materials at the module level that sit alongside the CBM outcomes and show how each outcome could be shown to be achieved by a student. Within each modality, we are providing teachers with tagged CBM outcomes so that they are aware of the CBM outcomes being either entirely or partially covered. In this way teachers can see direct examples of the CBM outcomes in situ and enable them gauge whether their students are achieving them. At the end of directed learning modules, we flip the context and ask a new question for the student to tackle independently in what we call a 'project'. In this manner they can show their understanding of the problem-solving process and the concepts and tools used within that problem context area. Each project is similar to the guided learning style deployment—a question followed by a scaffolded guide giving suitable questions to ask and prompts for where to take the solution.

In terms of assessment techniques, there are formative assessment opportunities throughout the directed learning style materials. We are currently building a bank of summative assessment methodologies that utilise the computer-based nature of the materials, from customisable individual datasets to free text analysis, to self-checking code solutions. This area is one of our major development areas for the next 12 months.

The silent C in STEM

Miles Berry; Andrew Csizmadia

Computing at School, England

Abstract

Computer science owes its very foundation to mathematics: 2016 marks the 80th anniversary of Turing's paper on computable numbers, with an application to the Entscheidungsproblem, and historically there have been very strong connections between these two domains. In England's new (2014) national curriculum, these connections are made apparent to primary and secondary pupils through an emphasis on mathematical reasoning in the mathematics curriculum and the 'golden thread' of computational thinking which runs throughout the computing curriculum. We explore how themes such as abstraction, algorithms, decomposition, generalisation and logical reasoning can be applied to solve problems in both domains.
Much of modern mathematics draws extensively on computation, and we discuss some ways in which computer software, and in particular programming in languages such as Scratch, Snap! or Python can be used to enhance and enrich learning in mathematics. We look too at the mathematical requirements of England's computing curriculum and consider how these can be addressed through 'unplugged' and computer-based pedagogies.
We conclude with a discussion of some of the ways in which the UK computing subject association, Computing At School, has supported the implementation of the curriculum, covering initiatives such as the Network of Excellence, Barefoot Computing and Tenderfoot Computing.

Introduction

2012 was a year of transformation for computing education in English schools. At that year's BETT Show, Michael Gove, then the Secretary of State for Education, announced that Information and Communication Technology (ICT) would be disapplied from the English national curriculum[1]. He subsequently announced that it would be replaced with a more robust and rigorous academic subject, for which statutory programmes of study were produced at the end of a lengthy consultation in 2012-13[2]. In the same year, the Royal Society published a report on computing education, Shut down or restart[3]. The report made a strong case for the inclusion of computer science in the school curriculum, as a distinct academic discipline, perhaps comparable to physics.

In describing the nature of computer science, the report's authors noted:

> *Computer Science is an 'underpinning' subject, in the sense that its concepts are useful to many other science and engineering disciplines, particularly physics, and in some cases they are relied upon to such an extent that they can be considered to be part of that subject too. For example, algorithms are sometimes considered to be an element of discrete mathematics, and the logical and rigorous approach of Computer Science has much in common with mathematics in general.*

But also that

[1] Gove, M. (2012). Michael Gove speech at the BETT Show 2012 (11/1/2012)

[2] Department for Education (2013). The National Curriculum in England. London: DfE

[3] Furber, S. (2012). Shut down or restart? The way forward for computing in UK schools. The Royal Society, London.

Berry, M. & Csizmadia, A.P. (2016) *The silent C in STEM.* [Working paper submitted for CIDREE-STEM 2016, December 22nd].

> *Computer Science can sometimes suffer from being assumed to be primarily a 'tool' for other sciences rather than a subject in its own right. It is both of these, and in particular it is a science and an engineering discipline. However most STEM initiatives do not explicitly refer to Computer Science as a STEM discipline.*

Hence, the somewhat Cinderella role of computing as the 'silent C' in STEM, unlike the more pronounced 'I' of MINT.

Computational thinking and mathematical reasoning

From primary school onwards, children's arithmetic has two quite distinct stages: thinking about the question, and then working out the answer. A sum as simple as 23 + 39 demands that the child be able to decode these symbols in some meaningful way, and determine which algorithm to bring to bear in order to calculate the answer: then, and only then, can the child go on to working out the answer. When faced with a word problem, for example, 'how much change will I receive from a 5 Euro note if I buy three apples each costing 40 cents?', the same two stages apply, this time demanding a degree of abstraction as the child moves from the particular context to its mathematical representation, in this case 5 - 3 x 0.40.

More generally, we might see that most, indeed perhaps all, mathematics mirrors these two stages - thinking about problems and then manipulating symbols according to rules (i.e. a more sophisticated version of working things out). The formalist view of mathematics is that it consists of the consequences of certain string manipulation rules: for example Euclidean geometry can be thought of as those statements which can be formed by manipulating geometric axioms according to the laws of inference. Even within this formalist paradigm, practical, useful mathematics demands some thinking about which particular manipulations of strings will take us towards the solution of the problem facing us.

This view of mathematics as symbol manipulation lies at the foundation of computing. Up until the 1940s, *computer* was a job title - given to those paid to do arithmetic on paper or mechanical calculators according to the rules and procedures given them by their managers. Turing[4] expressed this symbol manipulation view of mathematics in his seminal paper, 'On computable numbers, with an application to the Entscheidungsproblem', defining computable numbers as those which could be written down by a machine and generalising this to computable functions and computable predicates: it is on this work (and the parallel work of Alonzo Church[5]) that computer science is founded, and thus the strong connections between mathematics and computer science, from primary school level up, should not surprise us.

In the decades since Turing's and Church's work, mathematics, as with many other fields, has been transformed by digital computing. The symbol-manipulation (or working out) phase of the mathematics done in science, finance, social sciences, the arts and every other domain (other than education) is done now by digital computers, rather than by people, as Wolfram explains in his TED talk[6]. Indeed, much of the symbol manipulation of even pure mathematics is now often done by digital computers rather than mathematicians, or their research assistants, themselves (see, e.g., Appel and Haken's computer-assisted proof of the Four-Colour Theorem[7].)

[4] Turing, A. M. (1936). On computable numbers, with an application to the Entscheidungsproblem. J. of Math, 58(345-363), 5.

[5] Church, A. (1936). An unsolvable problem of elementary number theory. American journal of mathematics, 58(2), 345-363.

[6] Wolfram, C. (2010). Conrad Wolfram: Teaching kids real math with computers. TED.

[7] Appel, K., & Haken, W. (1977). Every planar map is four colorable. Part I: Discharging. Illinois Journal of Mathematics, 21(3), 429-490.

The first phase of mathematics - thinking about the problem or the system - remains largely unchanged. Creative and imaginative problem solving lies at the heart of mathematics. Polya[8] suggested four phases for problem solving: understand the problem, devise a plan, carry out the plan and look back, plus a number of associated heuristics. Wolfram[9] argues convincingly that this is what mathematics education should now focus on, given that actual computation is now done by machines, and suggests his own four-stage helix for problem solving: define questions, translate to maths, computer answers and interpret. There are parallels here with the development process in software engineering: specification, design, implementation and testing. It would be wrong to think of school mathematics as being confined to manual computation. Problem solving and mathematical reasoning is an essential part of mathematics education. For example, the English National Curriculum [10] aims to ensure that all pupils:

> *"reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language; and*
> *can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions."*

Given the strong connections between mathematics and computer science, it would be surprising if the sort of mathematical reasoning involved in understanding problems and planning their solution was not mirrored by similar thinking in specifying systems and designing solutions in the field of computing. Just as mathematics might be seen as thinking followed by symbol manipulation, so programming can be seen as algorithms plus code (cf Wirth[11]). Before programmers begin work on coding solutions, they need to have fully understood the problem and have a clear plan (an algorithm) for how to solve it.
The term 'computational thinking' has been coined to describe

> *the thought processes involved in formulating problems and their solutions so that the solutions are represented in a form that can be effectively carried out by an information-processing agent.*[12]

Whilst there is not yet a universal consensus over the exact ingredients of computational thinking[13], its importance in computing education is widely accepted. It is seen as a 'golden thread' running through the English National Curriculum for Computing[14], which begins:

> *"A high-quality computing education equips pupils to use computational thinking and creativity to understand and change the world. Computing has deep links with mathematics, science and design and technology, and provides insights into both natural and artificial systems."*

[8] Pólya, G. (1957). How to Solve It. Garden City, NY: Doubleday

[9] https://www.computerbasedmath.org/assets/img/case-for-computer-based-math-education/CBM_brochure.pdf

[10] Department for Education (2013). The National Curriculum in England. London: DfE

[11] Wirth, N. (1978). Algorithms + data structures = programs. Prentice Hall PTR.

[12] Wing, J. (2010). Computational Thinking: What and Why? http://www.cs.cmu.edu/~CompThink/resources/TheLinkWing.pdf

[13] Selby, C., & Woollard, J. (2013). Computational thinking: the developing definition.

[14] Department for Education (2013). The National Curriculum in England. London: DfE

Building on Brennan and Resnick's work[15] in which computational thinking is explored as concepts, practices and perspectives, Computing At School's 'Barefoot Computing'[16] continuing professional development programme for primary teachers identified six concepts and five approaches for computational thinking (qv Computing At School's QuickStart Computing guidance[17]). The concepts provide a unified approach to problem solving in both mathematics and computing, with a number of the example activities produced for Barefoot Computing linking these to topics in the English mathematics curriculum.

- Logical reasoning
  - In computing pupils use laws of inference to predict what programs will do from their source code, to detect and correct errors in algorithms and programs and to analyse the correctness and efficiency of algorithms; pupils learn about Boolean logic and its applications to circuits, programs and search. Program execution at the CPU level relies on logic gates.
  - Mathematics is underpinned by set theory and logic. Mathematical reasoning is fundamentally logical reasoning. In mathematics, pupils will be expected to 'show their working' and to provide a justification for their answer. They form a basic understanding of sets and their relationship, which is later formalised through Venn diagrams and the notation of set theory. They are introduced to simple proof techniques in Euclidean geometry, and will use reductio ad absurdum and induction at A level.

- Algorithms
  - From an early age, pupils learn about algorithms as sets of rules or sequences of steps for real life situations such as making a jam sandwich or tidying their classroom. They learn how other algorithms are implemented as code on digital devices. They learn that there are multiple algorithms for the same problem. They create their own algorithmic solutions to computational problems and are taught some classic algorithms for search and sort, finding greatest common divisors and testing for primality. They study greedy and divide-and-conquer algorithms in a range of contexts, including graph theory. They compare the efficiency of algorithms, in time learning to use big-O notation for this.
  - Pupils are typically taught standard algorithms for problems in arithmetic and subsequently algebra. This might be as simple as 'count out the first number of sweets; count out the second number of sweets; now count how many sweets you have' for integer addition, but will go on to include standard written algorithms for long multiplication and division, as well as methods for solving linear, simultaneous, quadratic and simple differential equations. Pupils are taught standard algorithms in other contexts, including testing for primality. They learn formulae for finding perimeters, areas and volumes, and for solving quadratic equations. Some pupils may discover their own algorithmic approaches to solving some classes of problems.
- Decomposition
  - Pupils learn to break down complex problems into smaller ones, tackling each of these in turn. Pupils learn how divide and conquer algorithms can be applied recursively, efficiently reducing the number of steps needed to solve a problem (e.g. Quicksort). Pupils make use of decomposition in their programming, using procedures, functions or objects to allow the different components of complex software to be developed and tested independently. At the hardware level, pupils come to recognise how digital

[15] Brennan, K., & Resnick, M. (2012). New frameworks for studying and assessing the development of computational thinking. In Proceedings of the 2012 annual meeting of the American Educational Research Association, Vancouver, Canada (pp. 1-25).

[16] http://barefootcas.org.uk/

[17] Berry, M. (2015). QuickStart Computing. Swindon: BCS.

devices are made of multiple, complex systems, each typically made from multiple, complex subsystems.
  - Decomposition is a powerful problem solving technique in mathematics, with pupils applying this in different contexts during their time at school. At elementary level, pupils recognise how numbers are decomposed into parts using place value, and subsequently prime factors. Simple arithmetic algorithms rely on ready familiarity with decomposition using place value. Pupils learn how the area or volume of complex shapes can be found through decomposition. Subsequently, pupils learn how vectors can be decomposed into orthogonal components and how matrices (and thus systems of linear equations) can be decomposed in a number of ways.
- Patterns and generalisation
  - In computing, pupils come to recognise standard ways to solve similar problems (for example, drawing equilateral triangles, squares and regular pentagons with a turtle), subsequently developing a general solution to a class of similar problems (in this case, drawing a regular polygon). Pupils learn to use libraries of functions developed by others rather than re-creating this code for themselves. They learn how other programmer's solutions to problems may be modified to solve similar problems. As pupils' software projects become more complex, they may make use of design patterns in their work, such as 'model-view-controller', which can be applied in a wide range of contexts from computer games to text editors.
  - Young children come to recognise patterns at an early stage in mathematics education, colouring in shapes according to a rule or deciding what will come next in a sequence. They generalise their own rules of conservation of number, shape and mass from observation. Later they are introduced to patterns in number, including the times tables as well as common sequences such as square, triangular numbers and the Fibonacci sequence. They conduct mathematical investigations, first describing the rules they discover and then expressing these in algebra, as recurrence relations and then as formulae. Pupils learn generalised algorithms or techniques - thus pupils learn the algorithm for long multiplication rather than memorising times tables to 100x100 or beyond and standard results for derivatives rather than computing each from first principles.
- Abstraction
  - For Jeanette Wing abstraction lies at the heart of computational thinking [18] and its particular form in compare science serves to distinguish computational thinking from other approaches to problem solving. Computer systems, both hardware and software, are so complex that computer scientists and software engineers established ways to *manage* this complexity, by hiding or setting to one side multiple layers of detail. Pupils might first meet the idea explicitly in the bottom of computational abstractions which model the state and behaviour of real world systems - for example the motion of a Snooker ball or the spread of an epidemic. They'll also recognise abstraction in functions, classes, libraries and APIs they use in their code: where the details of implementation are left hidden, and at times inaccessible, behind well documented specifications. They use abstraction in their mental models of computation (or 'notional machines'[19]) where the layers of user interface, compiler / interpreter, operating system and the processor's presentation layer sit between their actions and the copper and silicon of the hardware.

[18] Wing, J. M. (2008). Computational thinking and thinking about computing. Philosophical transactions of the royal society of London A: mathematical, physical and engineering sciences, 366(1881), 3717-3725.
[19] Sorva, J. (2013). Notional machines and introductory programming education. ACM Transactions on Computing Education (TOCE), 13(2), 8.

  – Abstraction is important in mathematics education too, with the curriculum taking pupils from the concrete to the abstract along a path that would be familiar still to Piaget. Even at an early age, pupils form an abstract notion of, for example, three-ness from the concrete three bears, three sweets, three books etc. They form an abstract notion of triangle or cube from the experience of particular triangles and cubes. In problem solving, pupils identify the important information in the phrasing of a question, setting to one side the less relevant or irrelevant detail. Algebra might be seen as an abstraction of number. Algebra, geometry, probability and calculus can be seen as the mathematician's approach to modelling the state and behaviour of real world systems.
- Evaluation
  – In computing, it's necessary for pupils to check whether the functions, classes and programs they write produce the results they should. It's also important that digital artefacts (including, but not limited to, programs) serve their intended purpose and are appropriate for their intended audience, and embody principles of good design. Pupils will also consider the efficiency, and indeed elegance, of their code.
  – In mathematics, pupils are taught to check their solutions, typically that numbers are broadly of the correct order of magnitude and make sense in the context of the original problem. They are also taught to check their working, that each step of their solution has been carried out correctly. Later on, they'll learn to look for logical flaws in proofs and perhaps even grasp something of the aesthetics of 'elegant' proofs.

The concepts of computational thinking can be learnt and applied in 'unplugged' activities, within and beyond computing, without the use of digital technology, as the above comparison with mathematics education illustrates, see also, e.g. CS Unplugged[20]. Wing's 'information processing agent' certainly includes digital computers, but need not be limited to such devices. That said, many would argue that 'computational thinking' can be developed particularly (perhaps most) effectively when linked explicitly to the 'computational doing' of computer programming:

> *Programming plays the same role in computer science that investigations do in maths or science. Programming animates the subject and brings computer science to life; it is creative, and engaging. It illustrates otherwise-abstract concepts in completely concrete terms. It is also an incredibly useful skill.*[21]

Papert wrote how he

> *began to see how children who had learned to program computers could use very concrete computer models to think about thinking and to learn about learning and in doing so, enhance their powers as psychologists and as epistemologists.*[22]

The experience of most primary and secondary computing teachers seems to be that computational thinking is best taught when linked directly with computer programming[23]. One could argue, as Wolfram does[24] that, if computer programs and computer programming were used more extensively in mathematics education, this would allow teachers and pupils to focus

[20] Bell, T., Witten, I. H., & Fellows, M. (2015). CS Unplugged

[21] Peyton Jones, S. (2014). Understanding the new programmes of study for computing

[22] Papert, S. (1980). Mindstorms: Children, computers, and powerful ideas. Basic Books, Inc..

[23] Taub, R., Ben-Ari, M., & Armoni, M. (2009). The effect of CS unplugged on middle-school students' views of CS. ACM SIGCSE Bulletin, 41(3), 99-103.

[24] Wolfram, C. (2010). Conrad Wolfram: Teaching kids real math with computers. TED.

much more attention on developing mathematical reasoning (or perhaps 'computational thinking') rather than mere calculation skills.

Using computers in mathematics education

There are many computer programs designed to contribute to mathematics education. The majority of these are one form or other of 'drill and practice' games, based on Skinner's behaviourist theory of learning[25], cycling through stimulus (question), response (answer) and reward (score). Well designed 'games' of this form are popular with both teachers and pupils and have their place in supporting the recall of mathematical facts and allowing pupils to receive immediate feedback on whether they have or have not correctly performed mathematical operations or solved problems. Some software of this nature includes an adaptive learning feature in which a pupil's response to one or more questions will determine which future questions are asked, allowing the stream of questions to be pitched appropriately at the pupil's current level of achievement.

Software tools are now available which allow teachers (and indeed pupils themselves) to create their own computer-marked quizzes, sometimes including adaptive learning features. Creating a quiz such as this is a good introductory programming activity, for teachers' professional development and as the focus for a unit of work in computing classes. Papert observed that creating drill and practice programs had more benefit than using them:

> *It is said that the best way to learn something is to teach it. Perhaps writing a teaching program is better still in its insistence on forcing one to consider all possible misunderstandings and mistakes. I have seen children for whom doing arithmetic would have been utterly boring and alienated become passionately involved in writing programs to teach arithmetic and in the pros and cons of criticisms of one another's programs*[26]

Beyond this, there are programs which can be used, very effectively, as tools for doing mathematics, or at least for the second, working out / manipulating symbols, phase of doing mathematics. These can, and many would argue should, be fully integrated into school level mathematics education. For example:

- Dynamic geometry software such as Geogebra allows pupils to perform geometric constructions, exploring how their constructions change as initial points, lines and arcs are changed interactively to discover for themselves the invariant properties of their constructions. Modern geometry software includes the ability to measure lengths, angles and areas, to perform transformations and to link geometry and algebra through plotting points and equations on Cartesian grids. The script for a geometric construction mirrors the steps of simple algorithms. The geometric objects constructed, and their properties and classes, can provide some insight into the foundations of object oriented programming[27].
- Computer algebra systems such as Mathematica provide extensive functionality for arithmetic calculations with arbitrary precision, working with algebraic expressions, performing differentiation and integration algebraically and carrying out computation on a far broader range of objects, including data, sound, images and video. Mathematica, and the associated Wolfram Language, can be a motivating and accessible way into functional

[25] Skinner, B. F. (1938). The behavior of organisms: an experimental analysis.

[26] Papert, S. (1972). Teaching children thinking. Programmed Learning and Educational Technology, 9(5), 245-255.

[27] Alberti, M. A., Bastioli, E., & Marini, D. (1995). Towards object-oriented modelling of euclidean geometry. The Visual Computer, 11(7), 378-389.

programming. Its creator Stephen Wolfram argues that it is a tool well suited to the development of computational thinking[28].

- The humble spreadsheet, for example Microsoft's Excel, can be used to visualise and summarise data, including randomly generated data in simulations, as well as to model mathematically the state and behaviour of complex real world systems. It too provides an introduction to functional programming through its system of declarative cell-based formulae, although it is a rather limited introduction as there is no readily accessible system of higher order functions[29].
- High end statistical data analysis packages such as R, allow pupils to move beyond the visualisation and summary tools available in Excel to explore data using multi-factor analysis, statistical modelling and machine learning, and to work with 'big' data sets which can provide a highly motivating context for learning that simple class or school based data collection activities cannot.

Coding in mathematics education

Given the connections between computational thinking and mathematical reasoning discussed above, and the likelihood that the former, and thus perhaps the latter, might be best developed through some practical experience of coding, it is reasonable to explore some opportunity for using computer programming in the mathematics classroom, in a way which not only allows pupils to develop their coding skills, but also provides some insights into mathematics.
Again, Papert's work provides some foundation for this:

> *"In my vision, the child programs the computer and, in doing so, both acquires a sense of mastery over a piece of the most modern and powerful technology and establishes an intimate contact with some of the deepest ideas from science, from mathematics, and from the art of intellectual model building."*[30]

Examples of programming activities in mathematics education might include:

- Turtle graphics: an approach to geometry in which the learner programs a 'turtle' (originally a small robot, now typically on screen) to draw geometric shapes. Initially these might be simple regular polygons, with the child learning for themselves how the sum of exterior angles totals 360°, but would later include compound shapes and recursively defined fractals such as Koch Flakes and Sierpinski Triangles. Turtle graphics was introduced initially in the Logo programming language, and this remains a popular introductory implementation, although most pupils are more likely to encounter this initially in Scratch's Pen commands. Secondary school teachers have found that Python's Turtle library provides a good means to bridge the gap from block-based programming in Scratch to more general purpose text-based programming languages[31].
- There is much more to both Logo and Scratch than turtle graphics. Scratch, for example, allows scope for pupils to create 'Monte Carlo' method simulations using random numbers, running Scratch in turbo mode to perform many trials of the experiment very quickly. A classic example is the estimation of Pi by comparing the distribution of random points in a circle to those in its bounding box[32].

[28] http://blog.stephenwolfram.com/2016/09/how-to-teach-computational-thinking/

[29] Peyton Jones, S., Blackwell, A., and Burnett, M. (2003). A user-centred approach to functions in Excel. International Conference on Functional Programming (ICFP'03), Uppsala, 2003.

[30] Papert, S. (1980). Mindstorms: Children, computers, and powerful ideas. Basic Books, Inc..

[31] Dorling, M., & White, D. (2015, February). Scratch: A way to logo and python. In Proceedings of the 46th ACM Technical Symposium on Computer Science Education (pp. 191-196). ACM.

[32] https://scratch.mit.edu/projects/61893848/#editor

- Primary and secondary mathematics places great emphasis on arithmetic using fractions, which are rarely supported directly in most common programming languages (Mathematica is a notable exception here). It's instructive for pupils to develop their own functions for fractions or to create a fraction class. The overloading of operators permitted in Python (or C++) allows pupils to then use their class as they would other numerical data types. One of the first tasks in developing a system for fractions arithmetic is calculating the greatest common divisors of two numbers: pupils can investigate for themselves alternate algorithms for such a task.
- Much is gained through pupils working out solutions to problems and puzzles through unplugged, pencil and paper approaches. Much too is gained through developing programs to solve such problems and puzzles, allowing a far broader class of related problems to be explored computationally. Perhaps the real distinction between mathematical reasoning and computational thinking is that in the former we seek a solution, in the latter a process through which the solution might be found. Project Euler is a rich source of problems that might be explored from mathematical and computing perspectives.
- Simple mathematical investigations, such as exploring how many different combinations of coins can make a set amount (as discussed by Graham et al[33]), can be tackled by young pupils through a process of trial and improvement or exhaustive search. Pupils can discover for themselves the recurrence relation in such problems, and may attempt to work out tables of values by hand, or as a spreadsheet. Such recurrence relations lend themselves to implementation as recursive functions in appropriate programming languages.

Mathematics in computing

Computer science itself makes demands on pupils' mathematical knowledge. The English computing programmes of study[34] were revised substantially in the drafting process at the request of the then ministers to increase the amount of mathematical content[35]. For 11-14 year olds, the new expectations include:

- *understand how numbers can be represented in binary, and be able to carry out simple operations on binary numbers*, which is often taught through 'unplugged' approaches in which pupils take what they know of decimal place value and arithmetic algorithms in base 10 and apply these to binary numbers and arithmetic in base 2. The standard algorithm for multiplication in base 2 is very similar to the 'Russian peasant' method for multiplication by doubling (ie a binary shift left) and selective addition, and binary long division offers a significant reduction in cognitive load compared to the equivalent base 10 version.
- *understand how data of various types (including text, sounds and pictures) can be represented and manipulated digitally, in the form of binary digits*: manipulation here might include shifting upper case to lower case by subtracting 32 from US-ASCII values, increasing or decreasing the volume of music by multiplying or dividing individual audio sample values, or brightening, darkening or reducing the colour depth of images through applying formulae to the bitmap. Pencil and paper exercises can be used here, but so can spreadsheets or text-based programming languages.
- *understand simple Boolean logic [for example, AND, OR and NOT] and some of its uses in circuits and programming:* pupils are introduced to truth tables; they also explore simple logic circuits, sometimes using online simulators, but often through pencil and paper exercises, recognising how binary addition can be accomplished through such circuits; the link between set theory and Boolean operators in search results is explored.

[33] Graham, R. L., Knuth, D. E., & Patashnik, O. (1989). Concrete Mathematics. Massachusetts: Addison-Wesley.

[34] Department for Education (2013). The National Curriculum in England. London: DfE

[35] Bannister, P. and Berry, M. (2013). Curriculum reform and computing. Presentation at Westminster Education Forum, 28/2/2013

- *understand several key algorithms that reflect computational thinking; use logical reasoning to compare the utility of alternative algorithms for the same problem*: pupils learn about algorithms for search and sort and, often, for some aspects of number theory, such as testing for primality or finding the greatest common divisor; they compare algorithms for time efficiency, although Big-O notation is typically left until elective qualifications at 16-18. A nice way to illustrate search algorithms is through playing 'guess my number' games; search algorithms are often taught using 'unplugged' approaches such as ordering weights using comparisons with a pan balance.
- *make appropriate use of data structures*: pupils learn about variables between the ages of 7-11. At age 11-14, they study lists and arrays: unplugged activities with lists might include different approaches to shuffling decks of cards, itself a useful topic for mathematical investigation. Some teachers choose to introduce elementary graph theory at this point, for example demonstrating how seemingly different problems may be represented using the same graph or setting pupils the challenge of finding a minimum spanning tree for a graph using a greedy algorithm such as Prim's[36].
- *understand a range of ways to use technology […] securely, including protecting their online identity and privacy*: some teacher use this requirement as an opportunity to teach pupils about cryptography, including public-private key systems, illustrating this with the relative difficulty of factorising large numbers compared to the ease with which prime numbers of a similar size can be multiplied.

Computing At School

Since its beginning in 2008, Computing At School (CAS, the UK subject association for computer science) has striven to support computing teachers in developing their subject knowledge, pedagogic practice and confidence in computing. CAS initiatives of particular relevance here are the Network of Teaching Excellence in Computer Science, Barefoot Computing, Tenderfoot Computing, QuickStart Computing, Project Quantum and CAS's Computational Thinking Working Group.

- The Network of Teaching Excellence in Computer Science (NoE), established in 2012, funded by the English Department for Education and administered by CAS, is a national community of professional practice. It draws its membership from teachers in schools and university academics, and focuses its support on the teaching of computing in state-funded English schools. In the period up to December 2016, the NoE provided over 56,000 instances of continuous professional development (CPD)[37]. These have been provided by CAS Master Teachers, university partners and CAS Hubs through formal training events including regional and national conferences, mentoring, coaching, peer observation, and peer collaboration in the development of resources and assessment. Since November 2015, ten university based NoE regional centres have ensured national coverage and consolidate and expand the network.
- The Barefoot Computing Project[38] began in 2013. It was initially funded by the Department of Education and subsequently by BT. Barefoot Computing took the view that primary computing teachers only needed to know enough computer science to teach the primary computing curriculum. It assumed that they knew already how to teach and how to use technology, but simply lacked computer science subject knowledge. It developed a set of concept guides to computer science topics, including a detailed treatment of computational

[36] Prim, R. C. (1957). Shortest connection networks and some generalizations. Bell system technical journal, 36(6), 1389-1401.

[37] BCS/CAS (2015). Network of Teaching Excellence in Computer Science DfE Project Quarterly Review December 2015. Swindon: BCS

[38] http://barefootcas.org.uk/

thinking as discussed above. These were supplemented by exemplar lesson plans in which primary teachers are shown how these computer science concepts could be linked to other topics in the English primary curriculum. For example, in the Bee-Bots 1,2,3 programming activity[39], pupils create an algorithm to draw the shape of a numeral, then they program a Bee-bot floor robot to navigate a route tracing the shape. The resources are freely available online, but are introduced to primary teachers via free workshops led by trained volunteers, including BT's own staff.

- QuickStart computing[40], funded jointly by Microsoft and the Department for Education, sought to develop quality continuing professional development materials for CAS Master Teachers and Hub leaders to use in meetings they organised, as well as providing resources on which primary computing coordinators and secondary heads of computing could draw for providing CPD for teachers in their schools. The primary pack focussed on computing subject knowledge, whilst the secondary pack focussed on issues of planning, teaching and assessing computing. Both packs included pointers to further reading, activities and resources.
- The CAS Tenderfoot[41] project began in 2015, funded by Google. It is a CPD programme for secondary computing teachers. It covers the background computer science subject knowledge, other than programming, that secondary computing teachers might need to teach the 11-14 computing curriculum, and, like Barefoot Computing, has a focus on the development and application of computational thinking. Topics available to-date include: bits and bytes, finite state machines and simulation. One approach used by Tenderfoot is that of puzzle-based learning in which puzzles are presented for learners to solve, learners' solutions are discussed and then the teacher presents a solution which leads to a discussion of the computer science or mathematical concept that underpins the solution. For example, the puzzle of the movement of a knight around a chessboard visiting each square only once can be solved by the use of graph theory and in particular the identification of a Hamilitonian cycle [42]. This project will provide high quality teaching resources and activities that teachers can adapt or adopt for their own pupils.
- Project Quantum[43]. Funded by ARM, Microsoft and Google, Quantum seeks to crowd-source a national item bank of multiple choice questions for assessing computing, including computational thinking. Some 2,100 questions are currently available, and as increasing numbers of pupils undertake questions, statistical (Rasch[44]) analysis of individual learners performance on each item will allow high quality questions and common misconceptions to be reliably identified.
- CAS convenes a number of working groups. These groups consist of subject matter experts drawn from computing teachers, educationalists and computer scientists. One of these groups, the Computational Thinking Working Group, sought to articulate, amplify and advocate what computational thinking is and provide advice and guidance for both preservice and inservice computing teachers on computational thinking and how it might

[39] http://barefootcas.org.uk/barefoot-primary-computing-resources/concepts/programming/ks1-bee-bots-12-3-programming-activity/ (free registration required)

[40] http://www.quickstartcomputing.org/

[41] https://www.computingatschool.org.uk/tenderfoot/

[42] Curzon, P. (2014). A Brief Tour of Computational Thinking: The Knight's Tour and Other Puzzles. London: Queen Mary Univesity of London.

[43] Oates, T., Coe, R., Peyton Jones, S., Scratcherd, T., and Woodhead, S. (2016). Quantum: tests worth teaching to. Cambridge: Computing At School.

[44] Rasch, G. (1993). Probabilistic models for some intelligence and attainment tests. MESA Press.

best be taught in schools. The group has produced a guide to computational thinking for teachers[45].

Conclusion

There is a symbiotic relationship between mathematics and computer science, at least to the extent that mathematics underpins computer science and computing can be used to automate much of mathematics. Beyond this, there appear to be strong parallels between computational thinking and mathematical reasoning: problem solving in either domain might usefully draw on a common vocabulary of concepts, such as that developed for some of CAS's projects.

[45] Csizmadia, A., Curzon, P., Dorling, M., Humphreys, S., Ng, T., Selby, C., & Woollard, J. (2015). Computational thinking: A guide for teachers.

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Changing mathematics education in Estonia: Computer-based statistics project

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Introduction

The accelerating growth of both quantity and availability of information afforded by new digital technologies has created greater expectations to people's competencies in numerical, statistical and visual literacy, requiring fundamental changes in teaching and learning. Mathematical modeling and processing of large data sets are commonly required skills for jobs in STEM sector, as well as in business and social sectors. According to the OECD Innovation Strategy, data analysis discoveries have become the drivers behind innovation. In the forthcoming years this will call for effective improvement of visual and statistical literacy of a large number of individuals. Currently, only 0.5% of the labor force in OECD countries are specialists working with data, but the request will remarkably increase in coming years (OECD, 2015)

In Estonia, the main goal of mathematics education is to teach students to formulate, employ and interpret mathematics in a variety of contexts, especially in different situations in their lives. The state final exams and the research evidences show that we have not yet achieved that goal. Whereas the performance of Estonian students in PISA mathematical tests is good (in PISA 2012, the average scores of Estonian students were ranked 10th to 14th among 65 countries worldwide, and 3rd to 6th in Europe), only 14.6% of students could solve complex mathematical problems that require mathematical modeling and well-developed reasoning skills (OECD, 2014).

Several studies have indicated that the shortcomings of contemporary math education is the capability to teach and develop skills for applying math outside the context of math lessons and specific textbook tasks. The traditional math education focuses at mastering of mathematical procedures within the context of mathematics itself. However, real-life problems require the ability to contextualize the problem in a way to analyze them using mathematical methods. Unfortunately, students still believe that math competencies are not needed in everyday life. The survey of graduates of Estonian higher education institutions in 2012 indicated that less than a half of these young people recognized the importance of STEM competences in their workplace (17% said it is very important and 24% important) (Laan et al., 2012). One can infer, that many employers do not believe in the usefulness of STEM competencies, and employees are unable to use their math skills efficiently in their jobs.

A survey of general competencies of Estonian lower secondary students (Palu, A & Kikas, E., 2015) revealed that students' procedural skills in mathematics are better than their problem solving skills. The study also examined the subject matter competence versus general math competence. The performance of students was notably higher within the math subject than in using general math competence to solve integrated problems.

The aim of this article is to introduce an innovative way to bring both the technology and the real-life problem solving experiences into the classroom. The computer based statistics project, that is currently being developed and implemented in Estonia, will be described and its positive aspects as well as challenges highlighted.

Computer Based Statistics (CBS) Project

The Estonian Ministry of Education and Research is financing the project on Computer Based Statistics, which aims at a fundamental change for learning data and statistics at lower and upper secondary levels. The first phase of the project in 2012 – 2014 created the preconditions for systematic innovation in mathematical education. The ongoing second phase of the project focuses on teaching CBS in a wide range of Estonian schools.

The new approach will empower students with the knowledge and modern skills for using mathematics and computers in real life (Wolfram, C., 2010). It is based on the innovative vision of Computer Based Math, introduced by Conrad Wolfram (CBM™, https://www.computerbasedmath.org/). A conceptually new curriculum and digital educational materials were developed for the secondary-level statistics course by experts from Wolfram Research, UK and from the University of Tartu, Estonia. The objective was to make the learning of mathematics more interesting to the students, improve their skills of mathematical thinking as well as improve their abilities to implement mathematical methods to real life situations. It also demonstrates how computers make the learning and real-life use of mathematics more diverse and effective. The project was supervised by the Project Advisory Board, consisting of academic experts, active-duty teachers, and representatives from public and private sectors.

The course includes lesson materials for 60 hours (25 at lower secondary and 35 at upper secondary level), and uses Wolfram software such as Wolfram Language, *Mathematica*, WolframAlpha and CDF-documents (Computable Document Format, usable with free CDF player). The number of hours and the main outcomes of the currently valid state curriculum were taken into account. However, the aims and the learning outcomes of the computer-based mathematics (CBM) curriculum are remarkably broader. Besides the specific mathematical skills, the CBM curriculum strongly aims at the achievement of general mathematics skills and transversal skills.

The new computer-based study materials were first piloted in the spring of 2014 with 40 teachers and more than 1800 students participating. The goal of the pilot was to test the suitability of such lesson materials, as well as analyze the attitudes and readiness to teach and learn in a nontraditional way. During the pilot, the reflections of teachers and students on both the course materials and the new teaching/learning method were examined along with its impact on students' performance.

Currently, the second phase of the pilot is taking place with another 40 teachers participating. The original materials have been improved and additional assignments with the necessary assessment criteria have been provided. Participating teachers have received additional training both in statistics and teaching with computers.

CBM curriculum and learning outcomes

Topics of probability, data and statistics are part of the mathematics curriculum in most countries. In Estonia, data and descriptive statistics are taught as a part of general mathematics courses throughout grades 5 to 9, and as a specific 35-hour math course in upper secondary level in grades 10 to 12. The upper secondary course also includes topics on probability and combinatorics along with the basics of statistical data analysis. Starting in 2011, the upper secondary course was enhanced with new sophisticated topics such as correlation, confidence intervals for the mean and reliability of statistical decisions among others. Students need these skills primarily for completing their own research project, which is mandatory for graduating high school.

When developing the new CBS curriculum, the number of prescribed hours and the main outcomes of the currently valid Estonian curriculum was taken into account. CBS curriculum covers a majority of mathematical concepts represented in the traditional curriculum. In addition, the new CBS curriculum expands upon important segments of working with data – e.g., how to collect reliable data using questionnaires, which data visualization methods to use, or how to estimate uncertainties of the results, etc.

The CBS curriculum supports context-based learning. It comprises of narrative-based modules concerning assorted real life problems. The math concepts and tools are learned in the course of problem solving. For example, in the first module "Am I normal?" students collect data in order to characterize themselves among their classmates. They use visualization techniques to learn about the empirical data distribution and its main characteristics. In an upper secondary module, “How many Estonian words do I know?“, students see how, using sampling, they could estimate the number of words they understand in a given language. Doing this, they first learn the basics of parameter estimation, then try to quantify how exact their estimation is (using confidence intervals) and finally estimate the right sample size to achieve the desired margin of error.

The learning outcomes of the computer-based mathematics curriculum are remarkably broader than those of traditional math courses. In addition to acquiring the required mathematical concepts and tools, the outcomes like “Abstracting to mathematics concepts”, “Designing and managing computations”, “Critiquing and verifying”, “Interpreting”, “Communicating and Collaborating”, etc. would be attained.

The study materials guide students to follow the investigative way of learning (see Fig. 1). A learning cycle includes four steps: the exploration of a problem and asking questions about it; mathematical formulation of the questions; mathematical calculations and visualizations; and lastly, interpretation of the mathematical results in the real life context. Preconditions for building a mathematical model, checking the reliability of the model, and consequences from the uncertainty of results are carefully examined. Hence, thinking about the problems is the priority. The objective is to understand the context, complexity and purpose of what is learned – and make the learning exciting.

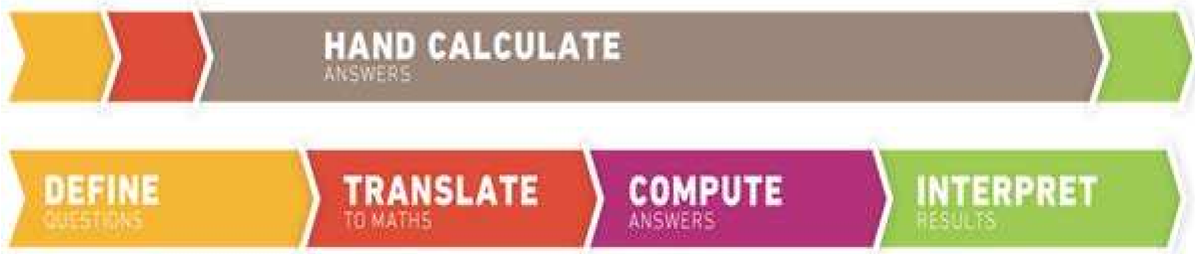

Figure 1. Illustration of the traditional and the CBM learning cycles

There are various learning techniques (modalities) used in the materials to diversify the learning process. Some modalities demand teamwork and cooperation; others support experimentation and creative practical tasks. Miscellaneous information sources and data visualization possibilities enhance interpretation and critical thinking skills, while formulation of written answers and writing essays develop self-expression skills.

Statistical data analysis methods have been advanced along with the development of computers and digital environments. It is natural to apply the concept of CBM at this time into this part of mathematics education. Using computers enables us to decrease the time spent on calculations and other mechanical operations, in order to gain more time for discussion and understanding. Using both students’ own data and data from the internet enables us to reinforce the learners’ interest in the subject and compose broader context around mathematical solutions.

Another goal of CBM is to embed programming into the mathematics education. The Wolfram language enables to implement calculations and sophisticated visualizations by short coding exercises, which is one of the means for making the CBM learning more efficient. The programming exercises included in the lesson materials will introduce students the possibilities of computerized mathematical modelling.

Digital lesson materials and learning process

The lesson materials were primarily created for regular school-based learning process where the teacher and the students are in the same classroom. Everyone has to use a computer with an internet connection. The lesson materials enable both digital communication between students and their teacher, as well as traditional verbal communication in the classroom.

Teacher's and student's materials have different screen views, whereas the teacher can also see student's view and show it on a screen. In addition, teacher's materials include ideas for activities, technical tips, recommendations for teaching, problem introductions, and (when available) the right answers to the questions. Both the student's and the teacher's materials have links to the theory files. The theory files are separate documents resembling Wikipedia articles. The theory files create theoretical background for the entire set of materials.

The lesson materials guide students to implement the consecutive steps of problem solving, illustrated in figure 1. Students can send their answers and opinions to their teacher, who can then summarize the student's answers, and give immediate feedback to the whole class. The teacher's main role is to direct the student's learning. Teachers should keep a balance between the context of the problem and the mathematical concepts needed to solve the problem.

The digital lesson materials have been created for a use in the Wolfram's *Mathematica* software environment and can be opened from a dedicated CBM menu. The lesson materials are also available as the Computable Document Format files (CDF files) applicable with the free CDF player. The materials in CDF-format enable all the interactivity described above; however, they cannot be changed without *Mathematica* software and one cannot complete the programming activities provided within the materials. Nevertheless, thanks to the innovation of Wolfram technologies, the new CDF-format materials now enable the programming exercises to be completed in the Wolfram cloud.

Teacher feedback

Each school in Estonia had an opportunity to volunteer for participation in the computer-based statistics project. The project leaders selected the schools who participated to be a representative sample of Estonian schools in terms of their sizes, location and previous performance on state math exams. Participating teachers were involved in the project starting at the very beginning of the first version of materials, and they received around 100 hours of training before piloting these new materials. The training included familiarizing the teachers with the CBM concept, discussing and practicing lesson materials, teaching with computers, and learning to program in the *Mathematica* language. In-depth training helped them to adopt the innovative educational approach, and conduct the pilot with high proficiency.

Throughout the pilot, all teachers continuously reflected upon the piloting process by sharing their experiences in the online forum, and filling in the questionnaires about learning activities and modules (Hommik, C. & Hoim, T., 2015) In return, they received technical and educational support from the project leaders and technical staff.

One vision behind the CBM curriculum is that all learning should take place in real-life context to be more meaningful and motivational for the students. This goal was well achieved and teachers appreciated that topics and tasks were creative, practical and in a real-life context. They also mentioned that this approach facilitated discussions with students in the classroom. However, some teachers were concerned about "learning too little mathematics". Their concern might have been focused around the state exams, since the CBS course does not drill typical tasks and definitions asked on the state final exams. Several teachers noted the lack of post lesson assessments, saying that by not having these assessments, it was difficult for them to get an overview of how much students had learned and how to grade them accordingly. To address this concern, the new version of CBS materials is now supplemented with extra practice problems at the end of each module. The first pilot also revealed the need for additional teacher training about contextual learning and application of statistical methods.

All participating teachers adjusted well to computer-based teaching owing to the teachers' overall positive attitude and experience of teaching mathematics with computers. The majority of Estonian math teachers use internet resources and specialty educational programs such as GeoGebra, Wiris, or the spreadsheet programs in their lessons. Participating teachers were able to overcome any technical difficulties in due course, and consistently maintained a positive opinion about technology-based teaching. The overwhelming majority of the teachers expressed their willingness to teach in the future with the CBS materials or combine CBS approach with the traditional curriculum. However, it also came out that they would prefer to have only half of the CBS lessons in the computer class. The reasons for this opinion are not clear yet, but hopefully the currently ongoing second pilot will clarify some of these questions.

Student feedback

The results of the first pilot showed that students assessed the new teaching approach and the learning materials highly. Various visualization possibilities, interactive charts, computer based learning, practical and realistic tasks, possibility to work in groups, and gaining new interesting knowledge were the issues more frequently mentioned as highly liked about the course. Interestingly, a smaller group of students marked the same items as disliked ones. The technical problems, working with charts, level of difficulty of the material, too much verbal reasoning, no new (mathematical) knowledge gained were mentioned as the most disliked aspects. The opposing opinions about these issues exhibit the diversity of attitudes towards innovation among students.

Similar to their teachers, student answers also revealed the imbalance between gaining the mathematical skills versus the context-based skills. Students thought that in the CBS lessons they mostly learned about the context-related issues (e.g. how happy are people in various countries), and did not clearly perceive the acquisition of mathematical concepts and procedures. Nevertheless, they successfully used those skills in the final test.

Students' attitudes and performance were measured before and right after the course. Students turned out to be more confident in their skills and more positive about their feelings (e.g., competence) when applied to statistics once they attended the CBS class. Tests given demonstrated improvement of knowledge and skills in both groups, the CBS group and the reference group taught in a traditional way. Unfortunately the final test administered at the end of the course paid insufficient attention on the extended learning outcomes and transversal skills achievable by CBM method and rather assessed the traditional mathematical skills. The final test results of the two groups were nearly the same (at 5% significance level) regardless of the fact that the CBS students enjoyed minimum workload as there was no homework and the course assessment was very liberal (mostly formative assessment in the classroom). It is therefore worthwhile to mention, that the CBS pilot group students achieved the same test results with less time spent on learning.

Only a small percentage of students (around 5%) were willing to tackle the programming exercises included in the lesson materials during the first pilot. Thus, only the most innovative students tried programming as a part of the statistics course. The most obvious reason could be that there was "too much innovation at once". Teachers also didn't emphasize programming because they already had to assimilate an entirely new teaching process. Since the context based learning is quite time-consuming, the average students did not have sufficient time for additional programming exercises. However, it is necessary to go on with the idea, and search for new ways to motivate students to link mathematics with programming.

Summary and conclusions

We believe that the CBM concept and the ongoing CBS project in Estonia are important first steps towards increasing the efficiency of mathematics education and improving application of mathematics in real life using digital resources. The CBS project has created a new curriculum and lesson materials, and tested their implementation in real school situation. The process has revealed a number of important issues, worthy of further discussion and research.

All participants in the project have praised the CBM approach, which enables mathematics education to evolve in line with the changes in the world around us. Teaching of data and statistics by computers is a natural step in math education and provides students with necessary skills for successful working in modern-day professions. The digital *Mathematica* environment facilitates efficient understanding of mathematical concepts by using data, formulae and visualizations. The various methodological tools (such as inquiry, experimentations, self-tests, strategy choosing, verbal reasoning, convincing, essay writing, etc.) support diversity of learning outcomes and develop both the mathematics and transversal skills.

In contrast to the traditional teaching, the CBS course places the mathematical tools into a broad context. It introduces different stages of the statistical problem solving process starting from data collection and verification, moving on to strategy selection and ending with the mathematical reasoning and contextual interpretation. Along with it, the CBS project brought up important curriculum issues concerning the role of context-based learning in mathematics. The following questions would emerge: Is it possible to acquire good mathematical skills in a problem solving course? What prerequisites in mathematics should students have for a successful contextual learning? What teaching methods would achieve the balanced outcomes in both the mathematical skills and contextual understanding? How to fit the more time-consuming contextual learning into the existing curriculum?

In a broader perspective, the question about the placement of the data and statistics course in a school curriculum is also worthy of discussion. Should the computer-based statistics course be a part of a math curriculum? Would it be reasonable to introduce an independent statistics course servicing the STEM but also the social subjects?

Overall, the teachers and students liked the CBM approach and the new interactive lesson materials. However, the generally positive attitude varied, depending on the lesson's theme, participating school and person. Most of the dissatisfaction of teachers was caused by different requirements in the valid curriculum and assessment compared to CBS. Fluent innovation in education assumes simultaneous changes in the curriculum, teaching methods and assessment. The inappropriate assessment criteria would remarkably lower the effect of innovative learning and teaching.

Piloting also helped to determine the requirements for technical equipment and support for CBS teaching. Good internet connection and up-to date server connections are crucial for teaching CBS at schools. The participating schools coped well with the installation of lesson materials in their schools. However, different operating systems with different versions caused additional complications. Ideally, the lesson materials should be accessible over the internet, which would decrease the need of technological support to schools.

Feedback from both the teachers and students will be taken into account for further development of the materials. The program advisory board has given their positive recommendation for step by step advancement with the CBS teaching materials to all schools in Estonia. The transition to using CBS in all schools will not be a quick process due to the fact that, in general, it is hard to give up long-time beliefs and values of traditional teaching and learning. In addition, the implementation of CBS will also require financial commitment to invest into school's infrastructure and well-functioning teacher training program.

The results from the first pilot indicate that computer-based learning was a success and necessary first step on the road of innovating classroom mathematics education overall. The currently ongoing second pilot is testing the improved course materials along with the post lesson assessments and additional mini-projects.

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Mathematics and STEM in Flanders

Patricia DeGrande, Lotte Milbou

Flanders

Introduction

The current core curriculum defines final objectives for mathematics and science in primary and secondary education. Transversal final objectives for ICT are being developed for primary education and the first stage of secondary education.

Currently a political debate about secondary education and the final objectives is ongoing in Flanders. At the same time, AHOVOKS is preparing a competence-based reform of the entire curriculum.

AHOVOKS (Agency for higher education, adult education, qualifications and scholarships) is in the process of developing frameworks for these different domains. The domains are inspired by the European Key Competences. Additional domains were also identified. The methodology is inspired by an existing framework, TOS21 (Technology at school for the 21st Century: a framework for technology education for pupils from 3 until 18 years). Based on TOS 21, the final objectives technology have been revised for primary education and 1st grade secondary education and has been implemented in schools.

For the future STEM education, the frameworks for the domains 'scientific literacy', 'mathematics', 'technology' and 'digital literacy' are relevant. Cross-curricular frameworks for 'learning strategies' and 'psycho-social development' are part of the major plan. The idea is to integrate subject-based frameworks and cross-curricular frameworks when developing future final objectives.

Current and legal situation

The current core curriculum defines separate final objectives for mathematics, science and technology for primary and secondary education. Transversal, cross-curricular final objectives for ICT are developed for primary and first stage of secondary education. In 2010, the final objectives in primary education and the first stage of secondary education for technology were introduced.

Simultaneously, cross-curricular final objectives were also introduced for Technical-Technological education in the second and third stage of secondary education. Whereas the attainment of final objectives is compulsory, schools have only the social task of striving for (and not attaining) cross-curricular final objectives with their pupils.

The current final objectives offer the possibility to integrate mathematics, science, engineering and technology in day-to-day contexts. A number of cross-relationships between disciplines are explicitly defined; more specifically, on the one hand between mathematics, science and arts, and on the other hand between technology and science.

The following final objectives are an illustration of these cross-relationships:

> 'The pupils can indicate the natural sciences as part of the cultural development and illustrate the interaction with society on environmental, ethical and technical level' (Natural science for the 2nd stage of general education)
>
> The pupils can indicate in concrete examples that science influences the choices within the technical process; (Technology for the 1st stage of general education)

However, there are no separate sets of final objectives in the current Flemish curriculum on how to achieve the integration in STEM.

The final objectives (as defined by the Flemish government) are "Minimum objectives the educational authorities consider necessary and feasible for a particular part of the pupil

population. Final objectives apply to a minimum set of knowledge, skills and attitudes for this part of the pupil population."
In other words, the Flemish government only defines on what pupils need to learn and not how they need to learn it.
Freedom of education has been included in the Belgian constitution. Mirroring the dual structure of the 19th-century Belgian political landscape, the educational system is segregated within a secular and a religious segment. This segregation results in the presence of multiple education providers. These providers each develop their own 'learning plans' to concretize and contextualise the final objectives in accordance with their respective mission, vision and pedagogical project.
As said before, the Flemish government only defines what pupils need to learn. The education providers define how pupils need to learn. The final objectives are formulated at a 'general' level, defining the concepts for different disciplines. The learning plans concretize those final objectives into learning goals. These learning goals will be linked to subjects, contexts, didactic approaches, etcetera.

STEM in Flanders

To address the shortage of STEM profiles in the labor market and to reduce the high levels of youth unemployment, the Flemish Government launched the STEM Plan of Action 2012-2020. The idea is to impact all levels of education and to link the key actors and actions.

The plan resulted in 83 STEM Academies in leisure time, 2 learning networks for schools and a STEM charter linking 300 companies and industrial partners. STEM didactics were developed for primary education and are under development for secondary education. Hundreds of projects emerged for schools and leisure time. A number of secondary schools took the initiative their curriculum for the 1st stage to offer their pupils an interdisciplinary STEM-module.

The STEM Action Plan started in 2012. Today, we see an increased enrollment in formal and informal STEM-related education. Although the plan is successful, there are still a number of issues to be addressed. Further implementing the STEM action plan requires a key focus on the Societal Impact of STEM (society in transition), the involvement of girls and SES-background of pupils. Although the high response to STEM-education is important for STEM linkages between education and the labor market, several schools often promote STEM only as a label, a marketing product. STEM becomes a part of their recruitment strategy. Profiling as a STEM-school allows them to distinguish themselves from neighboring schools.

Today, schools offer STEM in separate subjects (often in the science or technology subjects). The level of integration is not guaranteed. Due to the multiple initiatives at different types of education and the multiple uses of STEM on the school level, there is some confusion as to what STEM really entails.

To address this confusion, the STEM QUALITY FRAMEWORK was developed in 2015 to define STEM by in the '10 dimensions of Good STEM'.

1. Integration of the separate components of the acronym respecting the uniqueness of each component
2. Problem solving learning strategies
3. Skillful and creative research and creation
4. Thinking, reasoning, modelling and learning how to abstract
5. Strategic use and development of technology
6. Gaining insight into the societal relevance of STEM
7. Acquisition and interpretation of information and communication about STEM
8. Team work

9. Acquisition of 21st Century skills
10. Innovation

All ten dimensions can be linked to mathematical competences. Some speak for themselves: 'Problem solving learning strategies' are an essential part of the mathematics, 'Thinking, reasoning, modelling and learning to abstract' describes mathematical modelling and 'Team' as essential for the integration within STEM.

Modernisation of the education system in Flanders

Currently, a political debate about the structure of the secondary education and the final objectives is ongoing in Flanders. The different stakeholders (the public, education providers, pupils, industrial sectors, experts,..) are being consulted in designing this transition.
The new structure of the secondary education is designed based upon a screening of the current structure and the professional qualifications in the Flemish Qualification structure (EQF). In the future, graduating students in secondary schools will receive a certificate of qualification based on the final objectives in combination with the competences as defined in one or more professional qualifications (EQF).
A number of fields of study were identified as having a strong relation with STEM. These fields of study are in general education 'Mathematics/science', in technical education 'Technology sciences' and in vocational education 'Electrical installations'. Secondary education prepares students for higher education (professional or academic bachelor), for immediate employment in the labour market or serves both these purposes.
The level of mathematical competence as well as the mathematical concepts to be acquired, as reflected in the final objectives and professional qualifications (EQF), will be different for these different 'STEM'- fields of study.
At the same time, AHOVOKS (prepares a competence-based reform of the entire curriculum. AHOVOKS is in the process of developing frameworks for different domains. The domains are inspired by the European Key Competences. Additional domains were identified to cover blind spots in the European Key Competences (e.g. physical and mental health). The methodology is inspired by an existing framework, TOS21 (Technology at school for the 21st Century, a framework for technology education for pupils from three till 18 years). By the end of 2016, these frameworks should be finalised.
A framework represents the core components for a specific domain with respect to three different dimensions: understanding, handling, transfer.
'Understanding' entails building understanding in the domain, including specific knowledge and basic skills. 'Handling' describes the use and application. In 'Transfer', the indications to links outside the other domains and to day-tot day life are specified.

This framework is based on a review of scientific literature about the fundamentals of mathematics. It should be used for all levels of education, from nursery education up to the third stage of secondary education. Not all elements represented in the framework need to be included in the final objectives. The selection of elements will depend on the specific field of study and the age and the developmental stage of the pupil population.

To guarantee the implementation of STEM according to the 10 principles described earlier, the frameworks for science, mathematics and technology are highly relevant. Cross-curricular frameworks for learning strategies, digital literacy and psycho-social development are part of the competence-based reform of the entire curriculum. The idea is to integrate subject-based frameworks and cross-curricular frameworks when developing future final objectives.
For the third European Key competence (the mathematical competences and the basic competences in science and technology), three separate frameworks are under development: 'Mathematical competences', 'Scientific competences' and 'Technological and engineering competences'. The unique nature of the different disciplines (science, technology, engineering

and mathematics) lead to unique purposes, approaches, concepts and professions. To respect the unique nature of these different disciplines, we have developed three separate frameworks. In the current version of the framework 'Mathematical competences' the core components are titled 'Quantity', 'Space and (geometric) forms', 'Relationships and change' and 'Data and uncertainty'.

- The first core component 'Quantity' refers to the total of characteristics and principles with respect to numbers and numeracy, without any abstraction level.
- The second core component 'Space and (geometric) forms' is connected to geometry and focuses on the form, size, position of geometric objects and our visual perception.
- The third core component 'Relationships and change' emphasizes on the analytical an algebraic modelling of relationships and changes. The level of abstraction is more relevant for this core component than for 'Quantity'.
- The fourth component 'Data and uncertainty' can more or less be thought of as statistics and related subjects.
- To emphasize on the common, transversal character of a number of fundamentals objectives of mathematics, we put these concepts in the framework overarching the four core components and the three dimensions. There we find mathematical communication, reasoning and modelling.

When developing future final objectives for mathematics in STEM-related fields of study, this framework 'Mathematical competences', together in the frameworks 'Scientific competences' and 'Technological and engineering competences' and cross-curricular frameworks (digital literacy, citizenship, learning competences) will be used to assure the integration. In the dimension 'transfer: indication to links outside the discipline', the relation between mathematics, science and technology is made explicit to promote this integration.

Summary

When defining the mathematical competences and final objectives for STEM-related fields of study the following will have to be taken in account:

- Results of the parliamentary debate on final objectives
- Modernisation of the secondary education system in Flanders
    - Structure of the secondary education
    - Competence-based reform curriculum – frameworks for final objectives
    - Implementation of the Flemish qualification system (EQF)
- STEM- action plan – Quality frame for STEM
- Different stakeholders

Aldron, G. & Soury-Lavergne, S. (2016) *The new French curriculum for mathematics and technology*.
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The new French curriculum for mathematics and technology

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Context

French national curriculum has been deeply renewed since September 2016. Not only the teaching contents but also the organization of the school, from nursery school to K9, are concerned by the new reform, with a focus on interdisciplinary in the content and in the organization of the classes.

The mandatory education, from age 6 to age 16, are organized in 3 cycles of 3 years each:

- the aim of the second cycle (K1 to K3) is to make students construct the fundamental knowledge and competences;
- the aim of the third cycle (K4 to K6) is to consolidate the knowledge and competences in order to guarantee that each student is mastering the basic knowledge and competences;
- the aim of the fourth cycle (K7 to K9) is to deepen the previous learning in order to acquire knowledge and competences necessary to any French citizen; but also to make each student able to pursue studies and lifelong personal development.

The first cycle concerns nursery education, which is not mandatory in France, even if more than 95% of the 3 year-old pupils attend school.

The specificity of the new third cycle is to straddle the primary school and the lower secondary school (college in French), meaning that achievements in learning must be evaluated over the three years, in continuity between the two last years of primary school and the first year of low-secondary school.

The detailed French new curriculum which has been released in november 2015 and has to be implemented since september 2016, can be download here:
http://www.education.gouv.fr/cid95812/au-bo-special-du-26-novembre-2015-programmes-d-enseignement-de-l-ecole-elementaire-et-du-college.html

Intention of the curriculum

French curriculum is organized into two different frameworks, one focusing on the competences − the common core of knowledge and skills −, defining seven main key domains of competences and another focusing on subject matter to be taught, among which sciences and mathematics – the “program”. Teachers are supposed to organize teaching of each subject matter in order to make pupils develop the competences of the common core as well as each precise piece of knowledge defined by the program. The aim is to organize the teaching in a more coherent way, in which each subject matter, with its specificities, contributes to the development of students’ competences. Moreover, an explicit organization of interdisciplinary projects and interventions of teachers is scheduled in the agenda of the students. Therefore, it should favor a more integrated teaching of STEM, beyond subject matters divisions and toward more interdisciplinarity.

Another specificity of this curriculum is the introduction of computer science as a new teaching topic.

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The common core of knowledge and skills is divided into five main domains of competences, for which sciences, mathematics and technology contribute:

1. **Languages to think and communicate:** above the necessary development of the French language as a tool to understand and express the world, mathematics, computer science and sciences are also considered as providing languages that are useful to solve problems, to generalize solutions, to deal with data, to read and communicate results, to represent experiences, objects and phenomena... Therefore the “common core” encourages teachers to consider that mathematics and sciences are providing a language that every subject matter may contribute to construct and use.
2. **Methods and tools to learn:** using numerical tool and collaborating between students to drive a project are competencies that may be developed in every subject matter, especially in sciences and mathematics. Dealing with sources of information, questioning their relevance and validity, mastering digital environment to edit image, text, sound… are also the purpose of every subject matter. Moreover, using CAS, numerical data and computing software are specific to mathematics. Computer science taught in mathematics and technology allows to deepen the use of digital tools.
3. **Development of personal identity and citizenship:** the whole teaching content must contribute to the development of self-confidence and mutual respect. Mathematics and sciences contribute to the development of judgment, critical thinking and truth seeking, through the notions of argumentation and proof.
4. **Natural and technical systems:** observing the real world, in sciences but also in sport, art or media for instance, raises questions as well as the necessity to look for responses. Sciences, technology and mathematics treat those questions by providing powerful models and systems of representation. It provides fundamental tools to understand, to increase awareness and adopt a responsible behavior in the fields of environment, health and technological growth. In mathematics, knowledge about numbers, magnitudes and measurement of time and space, scales and proportionality, algorithms etc. are keys to understand the world and solve problems with different methods, including trial and errors, conjectures and validation. STEM is a major contributor to the development of competences of this domain.
5. **Representations of the world and human activity:** the objective is to make students build a personal and shared culture, also in STEM, by understanding the relationships between sciences, technology and societies. Their scientific and technological culture will help them to measure the effect and the sustainability of innovations.

STEM and teaching algorithms and programming

This new teaching doesn’t aim at training specialists of computer science, but aims at giving students decoding keys in order to understand the evolution of the digital world. It aims at acquiring competences and methods allowing to build an algorithmic thinking. The mastering of programming language is not an objective of this teaching, but their practices is a mean to discover methods and inquiry approaches as well as modelling and simulation skills.

Two approaches of computer science are rooted in two different epistemologies: a theoretical point of view rooted in mathematics epistemology (the idea of a computer instead of the computer as an artifact a legacy of Türing’s thinking) ; the modeling and abstraction coming from experimental sciences with semiotic approaches relatively to the translation but also conception of systems rooted in the engineering facet. As a consequence, the two different approaches are present in the curriculum, in the teaching of mathematics as well as in the teaching of technology:

- one more on the understanding of algorithms and their translation into a programming language (in fact most of the time Scratch or any other visual programming language).
  This teaching aims at developing the following competences:
  - analysing a complex problem and being able to split it up into sub problems,
  - pattern recognition,
  - generalisation and abstraction leading to structured and reusable instructions,
  - elaboration of algorithms using fundamental concepts of algorithmics as event-based programming, parallelism,...
- the other more on the possibilities of connecting objects (robot, connected objects,...)
  This teaching aims at using technological tools in two complementary directions:
  - study of technological systems aiming at understanding how a technological system is able to communicate with its environment,
  - data coding that makes easier to search the data, to make comparisons and to identify any patterns that require further investigation.

Following the interdisciplinary aim of the curriculum, these two approaches taught in two topics can meet in institutional time: two teachers working together on a single object of knowledge and bringing his/her own viewpoint. This constitute a transposition in school of a specificity of computer science, a "[...] *multifaceted field that encompasses scientific and engineering aspects which are manifested in algorithmic problem-solving processes, for which computational thinking skills and sometimes also artistic and creative thinking are required*" (Hazan & al., 2014 p. 24). This transposition is based on the particular competences promoted in the two disciplines.

In mathematics, the same six competencies, searching, modelling, representing, reasoning, calculating and communicating, are detailed for each cycle. Example for cycle 3:

- **Searching**. Solving problems by picking and organizing necessary informations from various supports: texts, tables, diagrams, graphics, drawings, sketches, etc. Being involved in a process, observing, questioning, manipulating, experimenting, raising hypothesis by using tools and already known mathematical procedures, by developing a reasoning adapted to a new situation.Testing and trying several solving paths. (Links with domain "Methods and tools to learn" and domain "Natural and technical systems").
- **Modelling**. Using mathematics to solve some problems of the everyday life. Recognizing and distinguishing problems belonging to additive situations, multiplicative situations, proportionality. Recognizing real situations that can be modelized by geometrical relations (alignment, parallelism, perpendicularity, symetrie). Using geometrical properties to recognize objects. (Links with domain "Languages to think and communicate"**,** domain "Methods and tools to learn" and domain "Natural and technical systems").
- **Representing**. Using tools to represent a problem: drawings, sketches, diagrams, graphics, expressions with parentheses,... Creating and using diverse representations of simple fractions and decimal numbers. Analysing a plane figure under different aspects (surface, outline, lignes and points). Recognising and using the first elements of code of a plane figure or a solid. Using and creating representations of solids and spatial situations. (Links with domain "Languages to think and communicate" and domain "Representations of the world and human activity**"**).
- **Reasoning**. Solving problems requiring to organize multiple dates or the elaboration of a process that combine steps of reasoning. In geometry, smoothly going from perception to a control by instruments to start reasoning only on properties of figures and relation between the objects. Evolving in a collective investigation by taking into account the others' points of views. (Links with domain "Methods and tools to learn", domain "Development of personal identity and citizenship**"** and domain "Natural and technical systems").
- **Calculating**: Calculating with decimal numbers, exactly a approximately, by using appropriate strategies and techniques (mentally, on a line or with computing algorithms).

Controlling the plausibility of own results. Using a calculator to find or check a result. (Links with domain “Natural and technical systems”).

- **Communicating**: using progressively an appropriate language and/or adapted notations in order to describe a situation or set out an argumentation. Explaining his/her own approach or reasoning, understanding others explanations and arguing

In technology, the competencies appearing in cycle 4:

- practicing scientific and technologic inquiries: formulating scientific question, posing hypothesis in order to solve a problem, elaborating experiences, using observation tools, interpreting results and drawing a conclusion, communicating both results and process, identifying and choosing adapted notions or models in order to enter a scientific process.
- designing and creating: being able to draw an experimental protocol.
- appropriating tools and methods: identifying and organising tools and techniques in order to keep a memory of research (orally or with paper and pencil).
- using languages: describing, using adapted tools and languages, the structure and the behaviour of objects. Applying the basic principles of algorithms and codes in order to solve problems.
- using digital tools: simulation of the behaviour and structure of objects, organizing and storing numerical data, reading, using and producing representations of digital objects, driving a connected system locally or remotely, modifying or parametrizing a connected object.
- adopting an ethic and responsible behaviour: improving good practices regarding connected objects and analysing environmental impact of an object and its components.
- self positioning in space and time: bringing together objects of a same family, linking technological developments to inventions and innovations that mark breaks in technical solutions.

Examples in the curriculum

We present below two examples extracted from institutional documentations, published by the educational council to illustrate the new program and support its implementation by teachers.

Solving problem with proportionality – Cycle 3

Solving problems of proportionality is a way to construct knowledge and develop competences in every domain of the common core. For instance, the necessity of verbalizing the procedures, like “taking the double, the triple... of a magnitude” helps to enlarge the lexicon and to deepen the understanding of mathematical notions. The study of the relationship between the two magnitudes involved in a proportionality problem needs efficiency in computing and the use of decimal numbers and fractions. Problems of real life, like prices, costs and reduction or augmentation and many problems of natural systems or technological systems can be modeled and solved with proportionality.

During the three years of the cycle 3, the notion of proportionality will be introduced through problems that can be written on the following models linked with different mathematical domains:

Number and calculation:

step 1: 8 times 10 equals 80 and 8 times 3 equals 24. As 13 is 10 plus 3, 8 times 13 is 80 plus 24.

step 2: 7 times 13 equals 91. As 35 is the quintuple of 7, 35 times 13 is the quintuple of 91, that is to say 455.

Length and measurement:

step 1: 5 kg of potatoes cost 6.4€ and 3kg costs 3.84€. As 5kg minus 3kg is 2kg, we deduce that 2kg of potatoes cost 6.40€ minus 3.84€ that is to say 2.56€

step 2: 500 sheets of paper is 3.5cm high. What is the thickness of one sheet?

Space and Geometry

step 1: situation of enlargement of a geometrical figure: rectangle, triangle...
step 2: situation of enlargement of a puzzle.
step 3: use of a map scale to determine the distance between two towns.

But the proportionality is present in numerous other school topics: geography, sciences and technology, sport, etc. as well as in real life situation (cooking, measuring,etc.). Teachers has to choose proportionality situations in the different domains and to teach students how to mobilize different procedures that can solve the different kinds of problems. In the same time and in order to make sense, situations which are not relevant to proportionality must also be exploited. Typically: if I measure 1m when I am 10 years old, I can measure 2m when I'll be 20 years old but surely not 4m at 40!
Proportionality table has not to be taught in itself but as a tool among others to solve proportionality problems.

Algorithms and programming – Cycle 4
We present here an example coming from institutional educational support for teaching computer science in mathematics. The initial maths problem is the following:

Four coins are lined up on a table. Clicking a coin flips it but flips also the coin on the right. Starting from Heads, Tails, Heads, Tails, how many clicks brings all coins on the same side?

In term of algorithm, this activity aims at working the following notions: conditional instructions, use of a variable, use of a list, event-based programming, parallelism, exchange of information between objects. It is also possible to generalize and simulate the problem with more than four coins and ask a more general problem: starting with *n* coins, is it always possible to bring them on the same side?

These two examples give an idea of the spirit of this new curriculum, the first showing how the mathematics skills can take profit of other topics to be taught and the second, despite the new topic, shows the preponderance of problem solving skills that have to be developed over years until the end of the French college.

Technology and computer science

Computer science can be seen both as a tool to teach and learn mathematics and as a body of knowledge to be learnt, in the courses of mathematics or technology. In both cases, digital tools are involved. Their role and place has to be questioned.
In this section, we develop two examples coming from our work, within research projects about mathematics education and computer science culture:

- OCINAEE Connected objects and Digital Interfaces for Learning at Primary School and problem solving in mathematics;
- Unplugged computer sciences

The issue of assessment in the STEM education is also developed under the scope of formative assessment relatively to the European FaSMEd project.

OCINAEE, a robot in mathematical games for primary school
The OCINAEE project – Connected Objects and Digital Interfaces for Learning at Primary School – aims to explore and design mathematics learning situations with a system of connected objects. The OCINAEE system is a set of interacting devices either tangible, like cards or dice, or digital, like tablets and smartphones. Connection between the two classes of objects is actuated by a mobile robot that can read physical elements such as cards or any printed material. The robot itself has properties of both worlds, the tangible one and the digital one. Like any concrete manipulative, it is an artifact with mechanical and physical properties,

mainly through its movements. However, it is also a digital artefact because its behavior results from instructions given by a digital environment either automatically generated or piloted by a user.
Four different games have been designed with the OCINAEE environment, addressing some main mathematical content for primary school like place value or addition of integers and decimal numbers and spatial knowledge and coding the positions and trajectories of a robot in a plane. The games have been designed to create problem solving situations.
We are currently trying to define the different kinds of actions and feedback either, tangible or digital, that such a complex environment should offer to support the students problem solving. For each possibility of action and feedback, designers have to choose between tangible or digital.

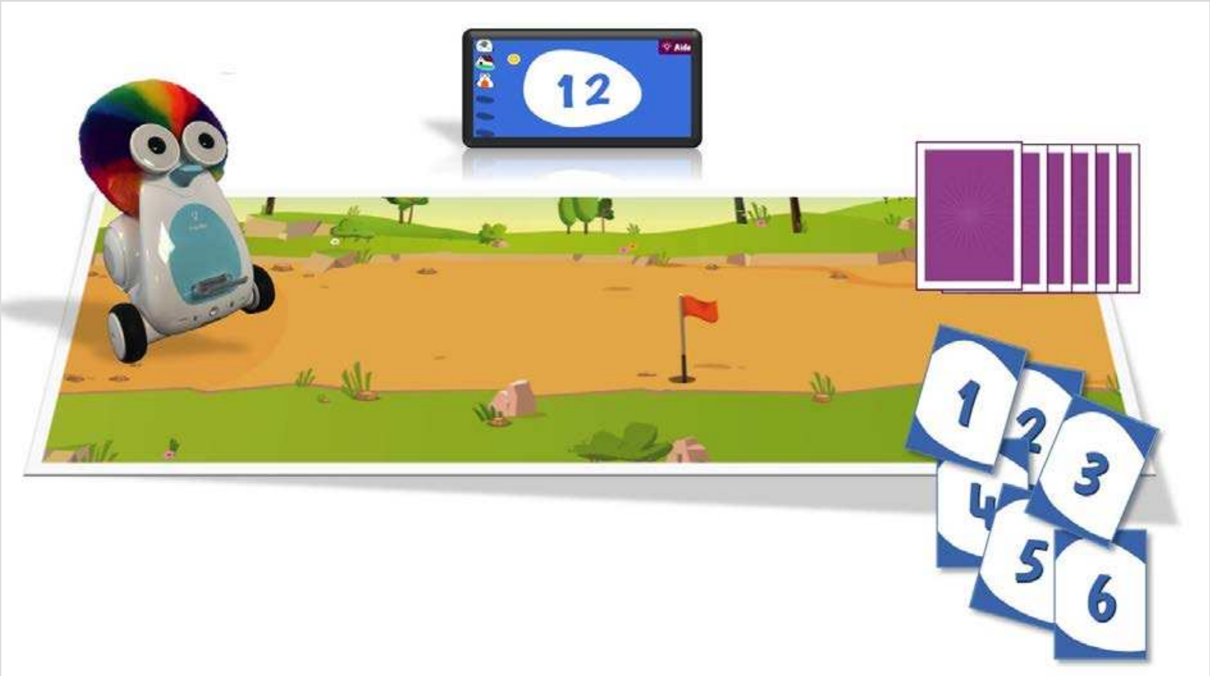

Figure 1. Some of the connected objects of OCINAEE project in scenario “the target number”: moving robot, smartphone displaying the target number 12, game board and sets of cards.

For instance, in a game named “the target number”, students have to choose 3 numbers out of 6 whose sum is the target number. In the current OCINAEE version, the target number is displayed on a smartphone and the numbers to be added are printed on cards with their symbolic writing. Students have to present cards to the robot. Then the robot moves on a line toward the target and stops before or after the target according to the sum. It also announces if the number of presented cards is correct or not. To implement such scenario with OCINAEE devices, we had to create direct manipulation feedback but also choose whether it is to be produced by a concrete or virtual object. For instance, the movement of the robot on the board is a feedback in the concrete space that informs the students about their answer (too small or too big), telling them something about their solving strategy. Moreover, even though it is a rather simple kind of feedback, this simple movement of the robot mediates the notion of number line. This feedback is also an evaluation, since reaching the target point indicates success. Another example of feedback is the fact that the robot’s eyes flash each time a card is presented but then, no indication of the numbers of cards already presented is revealed. This choice results from a didactical analysis: students need to know that the system takes a card into account but they have to deal with the fact that the result is a sum of three terms. They have to control it and they can succeed if they manage the cards and separate the ones they have already presented from the others. The tablet version of the scenario provides students' with virtual representations of numbers.

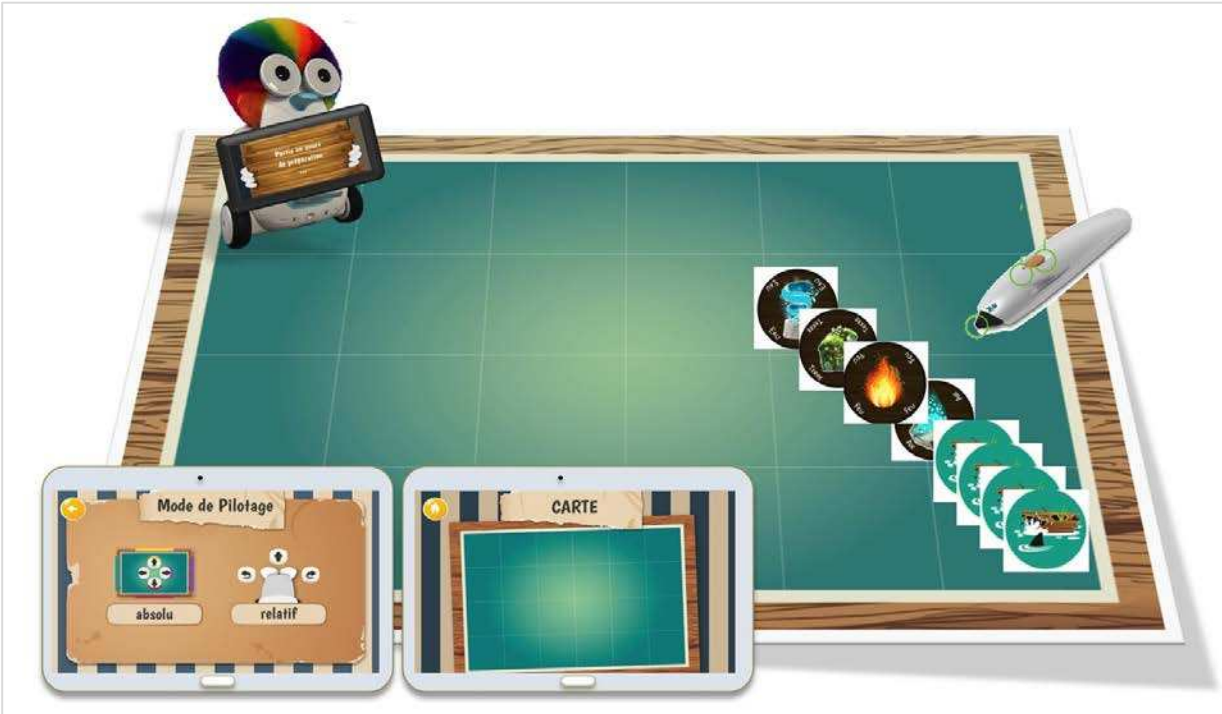

Figure 2. OCINAEE devices for the game "Journey in the plane"

In the game named "Journey in the plane", the students have to control the positions of the robot and of other elements on a plane. The plane is tangible, represented by a board that exist in two different versions: a squared plane and a virgin one. To drive the robot, students can choose between two ways to indicate the robot's direction: an absolute piloting device, that refers to the plane orientation (four different instructions, each being one step toward one of the four sides of the board), and a relative piloting device, that refers to the robot orientation (three different instructions, one step forward, turn to the right, turn to the left). It generates two ways to code the robot's trajectories and asks the students' to build relationship between the orientation of each devices (tablets, robot, board), even the board itself when it is virgin and the trajectory code. Therefore, the OCINAEE environment enables to design learning situations that combine knowledge related to spatial orientation and first element of coding. It provides an environment in which the model of a squared plane is a relevant tool to solve the problem of controlling the position and the trajectories of a robot.

New subject: computer science as a school subject

Unplugged computer science can be an opportunity to introduce an algorithm thinking without using computers. But it is also, at different levels, a way to make students think about algorithm, aside from any language constraint. The following example is in that sense typical of a reflexion about strategic games (or impartial game) where a player can always win. Starting from a chocolate bar whose one of the square is poisoned, and eating successively rows or columns, the loser will be the player that get the last poisoned square (fig. 3).

Students play the game and try to highlight a general algorithm allowing to win that is afterwards explained and institutionalized. The computer science principles that are illustrated by the game are linked to the graph theory and the concept of graph-kernel that can be extended to combinatorics problems. But also, it shows the possibility to solve a game using an algorithm that can be implemented in a machine.

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Figure.3. A dangerous chocolate bar

The aims of such activity is to increase the scientific awareness and to arouse intellectual interest to better understand the scientific approach and to provide a new outlook on why computer science is a science. With younger students, it is also possible to introduce the concept of algorithm using robots that can be programmed in a logo-way. Building activities with this kind of robots leans to think about mathematical concepts of space and of spatialization both with computer science concept of program.

FaSMEd, formative assessment in STEM

The aim of the FaSMEd (Formative Assessment for Sciences and Mathematics Education) project has been to investigate the role of technology in the formative assessment strategies for raising the attainment levels of low-achieving students in particular. For formative assessment we mean a method of teaching where:

"[…] evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited." (Black & Wiliam, 2009, p. 7)

We lean on the hypothesis that a technological environment can potentially support both the students and the teachers in getting and processing information about students' achievement in real-time. In particular, technologies that enhance connectivity and feedback can support the communication between teacher and students in such a way that students' productions can be collected, shown and shared in the classroom, and stored for further analysis and intervention. Students can inform their learning trajectories and teachers can make more timely formative interpretations and inform their future teaching. The way in which students and teachers can send and receive information deeply change. This form of assessment is really a starting point of any reflexion about assessment in STEM education. The objective of the project that was to develop sustainable teaching practices, that improve attainment in mathematics and sciences in a perspective of assessing for learning, brings us to outcomes that reveal educational opportunities for STEM education.

Aldron, G. & Soury-Lavergne, S. (2016) *The new French curriculum for mathematics and technology*. [Working paper submitted for CIDREE-STEM 2016, December 22nd].

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Conclusion

The main principles of this reform are presented by the institution with a will to renewal and inclusive pedagogy. The results of the evaluation latest Pisa and the need to change teaching practices including more technology are present in this process. Opponents of reform argue that disciplinary knowledge risk losing the weight against the educational organization. This year will certainly bring some interesting feedback that might appear as a productive research field and as an evaluation of this new curriculum.

Csapodi, C. (2016) *The remaining velocity problem with different solutions. Case Study.* [Working paper submitted for CIDREE-STEM 2016, December 22nd].
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The remaining velocity problem with different solutions. Case Study

Csaba Csapodi

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Abstract
The main aim of this article is to present different solutions to the so-called „remaining velocity" problem. I show three different ways to solve the problem apart from the "classical" way. During the past few years I've tried to get my secondary school students of different age and ability to solve the „remaining velocity" problem. It resulted the majority of students are able to solve the problem with the help of their teacher, but the task and the solution still were an eye-opener experience to all of them. This type of real-life based, cross-curricular, motivating tasks should be given more emphasis in the high school curriculum.

Introduction
In the past 50 years modelling and application have become part of mathematics education worldwide. The educational policy of the developed countries (OECD) urged the creation of PISA tests, one of the most popular and accepted international surveys in the field of mathematics, science and basic language-skills of 15 years old students. One of the fields surveyed by PISA tests is the development of mathematical literacy. "Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen."[46]
The problem examined in this paper is an interdisciplinary topic inspired by a real life event. By working on this problem modelling competency and aspects of mathematical literacy can be tested and developed. Solving the same problem by using more approaches is necessary in the development of mathematical reasoning (see [4] and [5]). Problem-solving in different ways develops mathematical knowledge and creativity in the students' mathematical thinking [4].
The Hungarian Mathematics curriculum aims to ensure that:

- students should have the ability and skills to be able to apply their knowledge in math;
- students should solve tasks independently, actively participate in the teaching and learning process;
- through the problem solving ability students can become precise, persistent, disciplined at their work;
- teachers can show by diverse examples, tasks the potential benefits in daily life, if a person is skilled in problem solving.

The problem is that few educational tools are available for mathematics teachers which can be used to achieve these goals. The project presented below is accessible in Hungarian language on the website of OFI (Hungarian Institute for Educational Research and Development, HIERD) website.[47]

[46] The PISA 2003 Assesment framework, p. 24, http://www.oecd.org/dataoecd/46/14/33694881.pdf
[47] http://ofi.hu/eletkozeli-gyakorlati-feladatok-0

Csapodi, C. (2016) *The remaining velocity problem with different solutions. Case Study.* [Working paper submitted for CIDREE-STEM 2016, December 22nd].

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Presenting and solving the problem

The most frequent reason for deadly public road accidents is fast driving, which is characteristic for whole Europe: according to a recent study speeding and/or inappropriate speed caused the 35-56% of severe crashes in the countries examined.[48] One of the main features of this phenomenon can be that drivers are not familiar with the consequences of fast driving. The majority of drivers ignore the risk of driving 50 km/h instead of the permitted 30 km/h in a protected urban area or 140 km/h instead of 130 km/h on a motorway.

Most students will acquire a driver's license at some stage of their secondary or tertiary education, so it seems important to deal with the above-mentioned problem during their studies of mathematics and physics. Solving the problem might have a shocking impact on students and consequently they may become aware and safe drivers in the future.

The idea of the next good practice experiment comes from Heinz Böer, the founder of MUED[49], and its detailed development is provided by Hans Humenberger, a professor at the University of Vienna.

The problem:

In a protected urban area where the permitted speed limit is 30 km/h, a car is being driven at a speed of 30 km/h. It is being overtaken by an other car, driven at a speed of 50 km/h. They are proceeding side by side when two children are running out from the pavement to the street to catch their ball. Both drivers start braking at the same time and both cars have the same brakes. The car with the lower speed can stop just in front of the children. What will be the remaining velocity of the car with the higher speed?

This is a fairly common phenomenon, and similar cases can occur not only in a protected urban area but also anywhere on the road.

Humenberger suggests two different ways for the solution (see [2]).

The first is a deductive approach in which the teacher guides the students step by step from the different units of speed measurement to the final solution of the problem. The author examines the problem using three test-sheets (A, B and C). In this case, the steps of the solution are as follows:

Test-sheet A

1. Converting the velocities into different units of measurements.
2. Through a concrete example constructing a formula determining the time required for stopping from velocity v_0 with deceleration b: $t_B = \dfrac{v_0}{b}$.
3. Calculating the braking-time: $s_B = \dfrac{1}{2} \cdot v_0 \cdot t_B = \dfrac{v_0^2}{2b}$.
4. Further discussion of the result (e.g. what can be said about the stopping distance in case of two or three times higher speeds).

Test-sheet B

[48] Study on Serious Road Traffic Injuries in the EU, European Comission, 2016 https://ec.europa.eu/transport/road_safety/sites/roadsafety/files/injuries_study_2016.pdf

[49] Mathematik Unterricht Einheiten Datei is the organization of teachers of mathematics in Germany. This material served as a basis for the DQME EU Project with the participation of 4 countries in 2004-2007 and 11 countries in 2007-2011.

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5. Through a concrete example plotting the distance-velocity function of braking from velocity v_0 with deceleration *b*.
6. Determination of the next function: $v(s) = \sqrt{v_0 - 2bs}$.
7. Further discussion of the result (as in the case of a body braking with deceleration *b*, what would be its velocity at the half of the stopping distance).

Test-sheet C

8. Presentation of the concrete example and the estimation of the remaining velocity.
9. Determining the remaining velocity by reading it from the plotted function or by substituting the given values to the following formula: $v_{\text{rem}} = v^*(s_B) = v^*\left(\frac{v_0^2}{2b}\right) =$
$= \sqrt{\left(v_0^*\right)^2 - 2b\frac{v_0^2}{2b}} = \sqrt{\left(v_0^*\right)^2 - v_0^2}$, where v_{rem} is the remaining velocity, $v^*(s_B)$ is the velocity of the faster car at the end of the stopping distance of the slower car, v_0^* and v_0 are the initial velocities of the faster and that of the slower car. (It is important to note that the remaining velocity does not depend on the deceleration of the cars.)
10. Completing the solution by taking into account the reaction time.
11. Additional investigations.

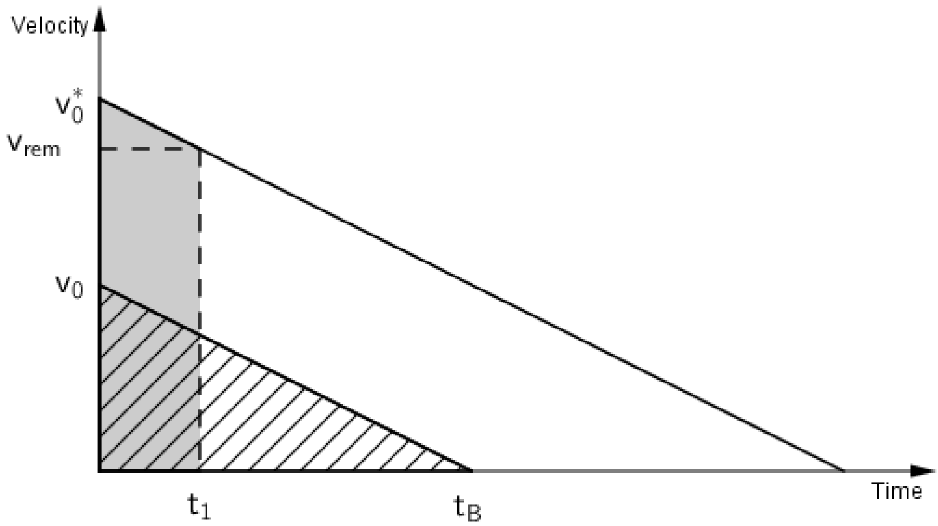
As an alternative solution, the author suggests a so called „throwing in at the deep end" method. Having presented the problem, the solution is entrusted to the students working independently, pairs or in small groups.

Further approaches to solving the problem

Succeeding, three solutions will be shown different from the above mentioned methods.

1. Geometrical approach[50]

This method suggests a geometrical approach on the basis of the graph of the time-velocity function instead of the distance-velocity function and solves the problem by determining the areas under the curves. Let's take a look at the time-velocity diagram of the two cars:



[50] Originates from Ödön Vancsó [3].

The distance covered by the slower car is equal to the area of the hatched triangle so the distance covered by the faster car should be the same. The distance covered by the faster car to the t_1 point of time is equal to the area of a trapezoid (represented in grey background). Based on this: $\frac{v_0^2}{2b} = \frac{v_0^* + v_{\text{rem}}}{2} \cdot t_1$. Since $v_{\text{rem}} = v_0^* - b \cdot t_1$, consequently $t_1 = \frac{v_0^* - v_{\text{rem}}}{b}$. Substituting these values in the equation describing the equality of the areas, multiplying both sides by 2*b* and arranging the equation, we get $v_0^2 = v_0^{*2} - v_{\text{rem}}^2$ and finally: $v_{\text{rem}}^2 = v_0^{*2} - v_0^2$.

This way the problem can be solved also by taking into account the reaction time of the drivers. In this case, the calculations become more complicated, see [1]. It is important to note if the reaction-time is also taken into consideration when calculating the remaining velocity, the result will not only depend on the value of the reaction-time, but also on the value of the deceleration.

This kind of problem solving requires students to understand why the area under the time-velocity function graph represents the distance covered by a body in motion. Besides, it transforms the problem into a geometrical one introducing students the complexity of mathematics.

2. The physical approach

A completely different way to solve the problem is the physical approach. The calculation is based on the employment of the kinetic and the braking energies. It is well-known that for the kinetic energy of a moving body with decreasing speed: $E_{kin} = E_{kin_0} - E_{break}$ where E_{kin_0} is the initial kinetic energy of the moving body. Given the fact that the two cars show the same braking performance and they cover the same distance, their braking energy must be equal. Furthermore, we know that the slower car will stop so its initial kinetic energy should be equal to the braking energy of both cars. Let's designate the initial kinetic energy of the slower car, the initial kinetic energy of the faster car and the momentary kinetic energy of the faster car respectively by $E_{kin\,a_0}$, $E_{kin\,b_0}$, $E_{kin\,b}$. Based on the previous thoughts: $E_{kin\,b} = E_{kin\,b_0} - E_{kin\,a_0}$ as $E_{kin\,a_0} = E_{break}$. Using the formula for the kinetic energy (and assuming that both cars have the same *m* mass): $\frac{1}{2}mv_b^2 = \frac{1}{2}mv_{b_0}^2 - \frac{1}{2}mv_{a_0}^2$ hence $v_b^2 = v_{b_0}^2 - v_{a_0}^2$ resulting the same formula for the remaining velocity as in previous approaches.

This solution has the advantage of emphasizing the relationship between mathematics and physics. However, its disadvantage is that it takes certain self-evident physical knowledge and formulas, the real content of which is not as believable as in the original case. To calculate the formula given for the kinetic energy is not so much different from the case when the distance-velocity function of a body moving with uniformly decreasing speed is described. It is important to note that this way the problem cannot be solved by taking into account the reaction time of the drivers. Therefore, this method is recommended rather as a curiosity than a unique solution.

3. Computer technique approach

Although this approach cannot be called a "solution", I have to mention it to calculate and present the data. One can display the relevant functions and the appropriate speed using for example the Geogebra program.

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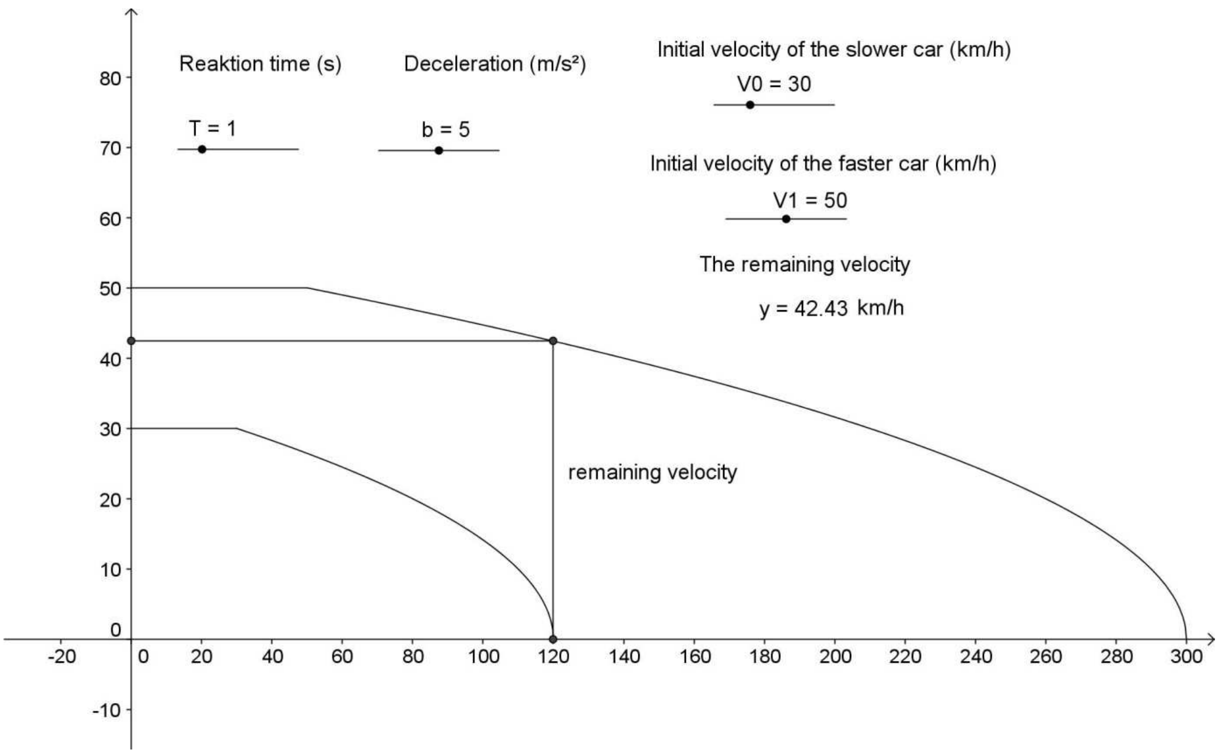

The changes of the velocities of the two cars in the function of the covered distance can be observed without or with the reaction-time. The input data (initial velocities, deceleration, and reaction time) can be chosen arbitrarily. The program calculates the remaining velocity for each case. Note that in this case the mathematical methods are embedded into the program so in this case some rudimentary mathematical knowledge is essential to understand the program completely. Even in this case it may be necessary to work out the formula of the distance-velocity function, but the plotting of the functions and the reading of the results can be achieved more quickly and accurately than by hand made diagrams. Therefore, this approach can be considered as a supplement to other methods.

Practical experiences

In former years I had the possibility to make experiments with students at Trefort Ágoston Grammar School in Budapest, whereby I tested two versions of the project and I've worked out a third method, accomplished also by the students (see [1]). Some experience is presented here shortly:

- It seems to be more appropriate if the students know clearly from the beginning (especially in the case of such a complex task) what the reason for the preliminary calculations is.
- "Deep end" way resulted that students of exceptional abilities are able to solve the problem independently, in pairs or small groups. The average or below the average students are able to solve the problem under teacher guidance.
- I have elaborated a third method for the steps of solving the problem: I started with the presentation of the problem and the estimation of the remaining velocities done by the students. Afterwards we were thinking together and I was continuously supporting them in the procession towards the solution, encouraging them to find it out themselves. That way the teacher became a moderator between the students and the problem.

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- I extended the concept with two other aspects: piloting a survey among the students and also engage their parents into the process. After having presented the problem to the students, I distributed the questionnaire and asked them to estimate the remaining speed in three cases. It was clear that students would underestimate the real values to a great extent. The vast majority of the students found the real values so absurd that they became interested in finding the reasons.
- Beginning the project I asked the students to take a close look at the results as they were expected to present the problem to their parents at home and ask them to complete a similar questionnaire. The main results were as follows:
    - The majority of the parents understood their children's presentation.
    - The majority of the parents declared that in the light of the results they would drive more carefully.
    - The parents found it useful when their children - among the subject matters of mathematics - also dealt with problems connected to everyday life.

Conclusions

The interdisciplinary problem solving inspired by a real life event has been examined in this paper and its assumption is that the remaining velocity problem can be adapted to secondary mathematics teaching. Although very few students were able to solve the problem on their own, nevertheless they were aware of the necessary mathematical and physical instruments. At the same time the majority of them were interested in the solution.

I am convinced that problems based on everyday life should be a more significant part of the secondary education.

Although the current Hungarian national curriculum in mathematics contains references showing the theoretical importance of cross-curriculum integration in teaching, but in practice they rarely appear. Neither the new textbooks nor the teacher education in Hungary contains sufficient resources to achieve these goals. It would be important to put more emphasis on contents as the example above during the ongoing renewal process of the national curriculum, the textbooks and the examination system.

Csapodi, C. (2016) *The remaining velocity problem with different solutions. Case Study.* [Working paper submitted for CIDREE-STEM 2016, December 22nd].

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Sutch, D. (2016) *Development of a STEM Course within the International Baccalaureate Diploma Programme*. [Working paper submitted for CIDREE-STEM 2016, December 22nd].
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Development of a STEM Course within the International Baccalaureate Diploma Programme

Deborah Sutch – Diploma Programme Mathematics Curriclum Manager

International Baccalaureate Organization

Abstract

All courses within the International Baccalaureate Diploma Programme are reviewed on a seven-year cycle from research, to development, to implementation. The current mathematics courses, of which there are four, are presently entering year five of this review process. It is proposed that this four course offering becomes two new courses, one with a traditional approach and one with the emphasis on applications, technology and STEM. The new courses will be ready for first teaching from August 2019, with the first examination in 2021.

This discussion paper aims to inform participants about the review process, the current courses, future plans, and in particular discusses the challenges and issues relating to the development of an applications and technology based STEM mathematics course, for students in their last two years of secondary education.

The IB's curriculum review process is a consultative, collaborative process involving many stakeholders, including universities, schools, teachers, IB graduates and experts. It begins with a comparison of international curricula and a review of current and anticipated trends in mathematics education. The presentation will review the findings of the research phase and discuss the implications of this for the IB Diploma Programme mathematics courses as we move in to the development phase of the review.

The uniqueness of mathematics within an international diploma programme will also be discussed and attendees will be able to find out more about the opportunities, challenges and issues that this gives rise to, including university recognition, pedagogical change, accessibility and assessment.

The IB Diploma Programme and IB Mathematics

In 1968, the IB Diploma Programme (DP) was established to provide a challenging and comprehensive education that would enable students to understand and manage the complexities of our world and provide them with skills and attitudes for taking responsible action for the future. Such an education was rooted in the belief that people who are equipped to make a more just and peaceful world need an education that crosses disciplinary, cultural, national and geographical boundaries. All of our programmes and courses are designed with our mission statement in mind,

"The International Baccalaureate aims to develop inquiring, knowledgeable and caring young people who help to create a better and more peaceful world through intercultural understanding and respect."

There are four IB programmes: the Primary Years Programme (PYP), the Middle Years Programme (MYP), the Diploma Programme (DP) and the Career-Related Programme (CP). This paper is concerned with the Diploma Programme which is a rigorous pre- university course of study designed for students in the 16 to 19 age range. It is a broad- based two-year course that aims to encourage students to be knowledgeable and inquiring, but also caring and compassionate. There is a strong emphasis on encouraging students to develop intercultural understanding, open-mindedness, and the attitudes necessary for them to respect and evaluate a range of points of view.

To fulfil the requirements of the DP, students are required to complete six courses of study, one from each "group" as outlined below. At least three of these courses must be at "higher" level

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(HL), each requiring 240 hours of study over two years, and a maximum of three at “standard” level (SL), which entails 150 hours of study over two years. Students must also complete a programme of creativity, action and service, a theory of knowledge course and within the two years write a 4000 word formal research paper known as the Extended Essay.

Figure 1

Mathematics is a compulsory subject within the diploma and the Mathematics group, known as Group 5, which currently contains Further Mathematics HL, Mathematics HL, Mathematics SL and Mathematical Studies SL. These courses are currently seen as being differentiated by mathematical aptitude.

Curriculum development in the Diploma Programme

All subjects within the Diploma Programme are reviewed on a seven year cycle which is internally defined. We benefit from being independent of political influences such as changes in government, and use a well-defined and structured process. The cycles are coordinated so that not all are implemented in the same year, so that schools are able to manage and adapt to changes.

The process of review is spread across three phases; these are evaluation and research (2 to 3 years), development (3 to 4 years) and implementation (2 years). These three phases can overlap depending on the nature and impact of the outcomes of the evaluation and research phase.

DP Mathematics (Group 5) is currently beginning year five of its review cycle, so we are in year two of our development phase.

Figure 2 presents more detail as to the activities during the three phases.

Sutch, D. (2016) *Development of a STEM Course within the International Baccalaureate Diploma Programme*. [Working paper submitted for CIDREE-STEM 2016, December 22nd].

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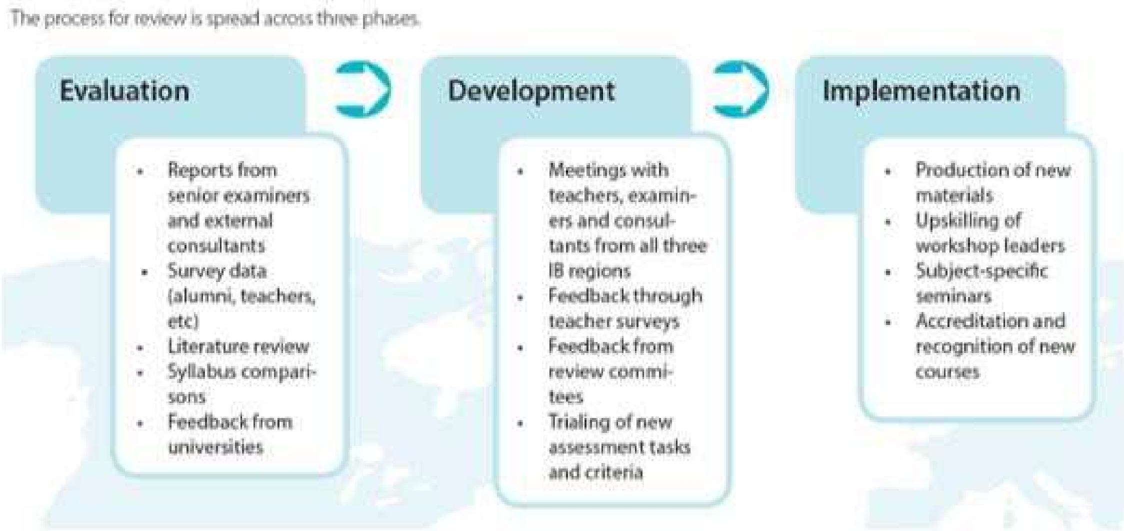

Figure 2

Current Mathematics curriculum review

Mathematics is currently in the development phase at the beginning of year five of the seven year cycle. Two new courses are now being developed which will replace the four current courses. Both of the new courses will be offered at HL and SL, with the SL component being a complete subset of the HL course. One of the courses will be what might be thought of as a traditional mathematics course or a "pure" mathematics course. The other course is more applications and contextually based with an emphasis on the use of technology, this could be thought of as an "applied" or STEM course. Students will be able to choose their course and level dependent on their own aptitudes and preferences, bearing in mind the requirements of any university course of study for which they may be applying.

Both courses have the same five "domains" which are algebra and number, functions, geometry and trigonometry, statistics and probability, and calculus. There is a "core" 60 hours in common between the courses to ensure the parity of the courses, however the emphasis within the domains is different depending on the course. For instance, the applications and contextual course has a far greater statistics component than the pure course and the pure course has a more extensive calculus component than the applications course.

Developing two courses at the same time raises particular issues and there are also some important questions that arise during the development of a STEM mathematics course.

i. Demands of the course as a component of a whole diploma program

   The demands of a course being developed as a component of a whole diploma program must be considered. The time commitment and intellectual challenge must be in line with other subjects within the diploma and content overlap needs to be very carefully considered. For example, students within the diploma can take a course in Computer Science where they are able to learn coding. Our STEM course would like to contain an element of coding, but how we make this uniquely mathematical so that it does not overlap with Computer Science has been an interesting discussion point.

ii. Accessibility for the least able and challenge for the most able

As mathematics is a compulsory element of the diploma we must ensure that the STEM course is accessible to those who in many other educational environments would not have chosen to take mathematics after the age of 16. At the same time the course needs to be challenging enough for those students who may wish to continue with this type of mathematics at university.

iii. University recognition

University recognition is an area that has been discussed from early in the course development. A new STEM course will need very careful promotion with universities in order that the mathematical value of the course is recognized when compared to more traditional mathematics courses. One university recently commented that although a STEM course might not fulfil the requirements for a first year undergraduate programme the benefits of this type of course later in an undergraduate or even post graduate education would slowly reveal themselves.

iv. “Assessibility” of the content

There are two challenges arising for us with the assessment of a STEM course. The first is related to the method of assessment for these kind of skills and the second is related to how universities and employers will regard this type of assessment. We are hoping to make extensive use of on-line assessment with students being expected to be able to use spreadsheets, dynamic geometry, a coding language, and a graphing package, however this affects the nature of the type of questions that can be asked and is very dependent on available technology. We visualize having a case-study or database oriented problems which requires a new kind of expertise in question setting at this level. Whilst many STEM activities represent beautifully enriching and engaging opportunities, the formal assessment of them in a “high- stakes” qualifications environment is a risky endeavor.

v. Current trends in mathematics education

Whilst there is much talk about exciting new developments in mathematics education at this level most mathematics courses still reflect a high degree of conservatism. If we take for instance ideas such as algorithmic thinking or statistical literacy, while the idea that these are useful mathematical skills is becoming more popular, actual instances of courses teaching these explicitly are isolated.

vi. The increasingly rapid development of technology

At ICME13 this year Jeremy Roschelle presented a lecture entitled “Technology for learning mathematics: what can we learn from large-scale studies”. He spoke about the fact that graphical calculators are still widely used in mathematics classrooms for teaching and learning and with the advances in technology why this should still be so. In essence their ease of use, short learning curve, portability, extendibility and price are important factors. When choosing which technology to incorporate into a course we have decided to consider the mathematics first and then choose the technology, rather than choosing the technology and looking at the mathematics. This means that many different types of technology can be used or recommended to our teachers.

Sutch, D. (2016) *Development of a STEM Course within the International Baccalaureate Diploma Programme*. [Working paper submitted for CIDREE-STEM 2016, December 22nd].

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International trend analysis – how and on what basis trends are identified

This is carried out in in many ways from formal to informal and takes account of qualitative and quantitative data. In general the international trend analysis will take place and be reported upon formally at the end of year one of the review cycle. This report forms the basis of a number of documents which are considered at the first meetings of the curriculum review team to decide the direction of the review.

Information on international trends comes from a variety of different sources and the Curriculum Manager, in liaison with our Global Research Department, is responsible for evaluating and reporting on the importance of the sources so that collaboratively the direction of the review can be determined. The following list details the usual sources of information which are required, however this list is not exhaustive:

- reports from Senior examiners
- commissioned reports from external agencies
- “expert” panel discussions
- surveys of alumni, teachers, universities
- literature reviews
- conferences
- focus groups
- syllabus comparisons
- feedback from universities
- feedback from IB teachers using our Online Curriculum Centre (OCC)

For the current review, syllabus comparisons were made between Finland, Massachusetts, Scotland, Ontario, The Netherlands, Mexico, Germany, England, Chile, Singapore, New Zealand, and Victoria and Queensland in Australia.

As an international organization with teachers and schools located throughout the world, the IB recognizes that it is in a unique position with regard to access to information on international trends. Many of our teacher educators consulted as part of the review are active within the IB and also within their own local mathematics communities.

Conclusion

The IB follows a very structured path when carrying out the seven year curriculum reviews. IB Mathematics has decided to develop two new courses for first teaching from 2019. By careful analysis of the current courses and by identifying international trends it has been decided that one of these courses should be applications, contextual and technology led.

This analysis and development raises many issues however the IB is now committed to developing this course.

Linney, R. & Walshe, A. (2016) *Solving problems with maths*. [Working paper submitted for CIDREE-STEM 2016, December 22nd].

Solving problems with maths

Rachel Linney; Anna Walshe

National Council for Curriculum and Assessment - Ireland

Introduction

In recent years, a consistent finding across national and international tests of attainment in mathematics is that Irish primary and post-primary students find difficulty with items assessing higher-order thinking skills e.g. modelling, abstract reasoning and problem solving(Shiel, Kelleher, McKeown, & Denner, 2016) . There is a growing concern that relatively few Irish students are performing at the *advanced level of proficiency* in mathematics(Mullis, Martin, Foy, & Arora, 2011) (Perkins, Shiel, & Merriman, 2013). Added to this, national reports draw attention to low levels of computer usage by students in Ireland and highlight that even when computing is used in education, it is mainly for low-level activities such as word processing, internet searches and playing computer games (DES, 2011). Limited use is made of computing in the development of higher-order thinking skills, creative or collaborative skills, independent working skills, or communication skills (DES, 2008); (Lau, 2009); (Eivers, Close, & Shiel, 2010).
In 2005 the NCCA published a paper *Review of Mathematics in Post-Primary Education* (NCCA, 2005) which pointed to a range of troubling realities about mathematics education in Irish post-primary schools. Against this backdrop and informed by commissioned research *International Trends in Post-Primary Mathematics Education* (Conway & Sloane, 2006) an ambitious reform initiative *Project Maths* was proposed. The aim of the *Project Maths* initiative was to reform mathematics classroom practice and in doing so, improve students' experiences and attitudes towards mathematics whilst at the same time raising their mathematical attainment. With its emphasis on skills development, reasoning and problem solving, the Project Maths initiative has started to change how students engage with mathematics in the classroom. For this reason, and to address some of the concerns mentioned at the start of this paper, it is timely that the Irish Leaving Certificate Applied Mathematics curriculum is reviewed and aligned with these developments.

Reform of Applied Mathematics

Written over 40 years ago, the LC Applied Mathematics syllabus (Appendix 1) mirrors a section of the LC Physics syllabus known as Mechanics. Over the years, possibly because of the content overlap between it and LC Physics, the subject has become a popular option for male students taking Higher level Mathematics and Physics. Although Leaving Certificate subjects are prepared for a 180 hour course of study, a practice has emerged with Applied Mathematics where students rarely study the course for 180 hours. Most students' study the subject either in school 'off timetable' or with a private teacher outside school; their experience of learning in Applied Mathematics is often reduced to exam preparation.

> *The only way to get good at Applied Maths is to practice as often as possible and as many different questions as you can get your hands on. To help you with this, the school has a stock of Applied Maths papers going all the way back to the 1970s (School website)*

With the examination regarded as predictable,

> *Almost every question which appeared in recent years was similar to at least one other question on an older paper – the natural conclusion being that if you cover all the old papers along with the recent ones you really*

> *should see very little new material in the leaving cert exam. (The Physics Teacher website https://thephysicsteacher.ie)*

Chosen by less than 3% of the Leaving Certificate cohort annually, less than 500 of whom are girls, approximately 90% of Applied Mathematics candidates sit the examination at Higher level, and more than 25% of these students achieve an A grade[51].

What people had to say about the direction for Applied Mathematics in Ireland

A public consultation on the future of Applied Mathematics ran from November 2014 to December 2014. The aim of this process was to hear the views of the public and the stakeholders on the future structure and content of Applied Mathematics at upper second level. The response delivered a consensus that the new Applied Mathematics curriculum should:

- emphasise deep mathematical learning
- develop their synthesis and problem-- solving skills.
- provide students with the multiple and varied opportunities to consolidate their understanding of LC Mathematics
- include applications of mathematics to solve real world 21st Century problems in motivating and exciting contexts
- support teachers in reconceptualising mathematics, teaching and learning
- Provide multiple, diverse, valid and robust opportunities for students to achieve
- Embrace technology and collaboration in the learning teaching and assessment associated with the specification

The response from the consultation echoes wider concerns internationally about the place of mathematics and skills in the workplace. It is generally acknowledged that if a job can be distilled down to a set of tasks that it can be replaced by a machine. In other words, if you can write down the rules of what you do then your job can be automated. The new course, therefore, should enable students to further develop the uniquely human capacity to reason in unstructured environments There is considerable evidence that working with authentic data enables students to understand the relevance of mathematics to everyday life, gain insight into how people use mathematics, and develop their capacity to be critical consumers of mathematical information (Kolsto, 2001). Despite this evidence, and teachers' acknowledgement of the fact that the ability to make sense of data is important, teachers and students do not make the connection between mathematics that is studied for an examination and the mathematics of everyday life. They tend to exclude real, messy, data from problem-solving as it introduces a level of uncertainty that is not welcomed by students and distracts from the focus on content (Albe, 2008). Although data is certain, the social and global situations that is describes are uncertain, and rarely have a 'correct' answer (Ratcliffe, 1997). Uncertainty is difficult to cope with in a performative teaching culture and so is avoided wherever possible. It is hoped that the revised Applied Mathematics specification will provide a vehicle for mathematical uncertainty that is set in a structured format so that students can learn how to deal with authentic problems.

[51] Statistics taken from https://examinations.ie

The evolution of the new specification

Following the consultation, the curriculum development group for Applied Mathematics agreed an aim for the new specification:
The aim of the revised specification for Applied Mathematics is

> *to develop the learner's capacity to use mathematics to solve real-world twenty-first century problems. By focusing on all aspects of the problem-solving cycle it is envisaged that learners will see beyond calculating procedures and gain experience in asking appropriate questions, formulating mathematical representations of problems, and interpreting and verifying results. Through this programme students should learn to appreciate the extent to which mathematics is relevant in everyday life, and this should generate engagement and interest. It is anticipated that computers will be used as a learning tool in some aspects of this programme.*

A specification based on mathematical modelling aligns closely with this aim. Mathematical modelling lends itself to a problem centric structure. Models are used across all areas of our society and problems that relate to students' lives can be motivating. Viewing global issues through the lens of mathematical models places mathematics at the core of young people lives, influences their decision making and presents them with the big picture of global issues as well as equip them with problem strategies that they can use in their immediate lives. The concept of *mathemacy* as a parallel to *literacy* has been used in which the societal role and function of mathematical modelling in the teaching of mathematics creates an important motivation for learning mathematics and modelling among students (D'Ambrosio, 2016) (Shear, Gallagher, & Patel, 2011). This seems to be especially evident in countries where there is poverty and inequality (Skovsmose & Valero, 2005). Empowering students to use mathematical modelling to reflect critically on societal issues and to criticise specific mathematical modelling processes and authentic applications of mathematical models in real life situations, is therefore pinpointed as an important goal for teaching mathematical modelling and mathematics in general under the socio-cultural perspective.

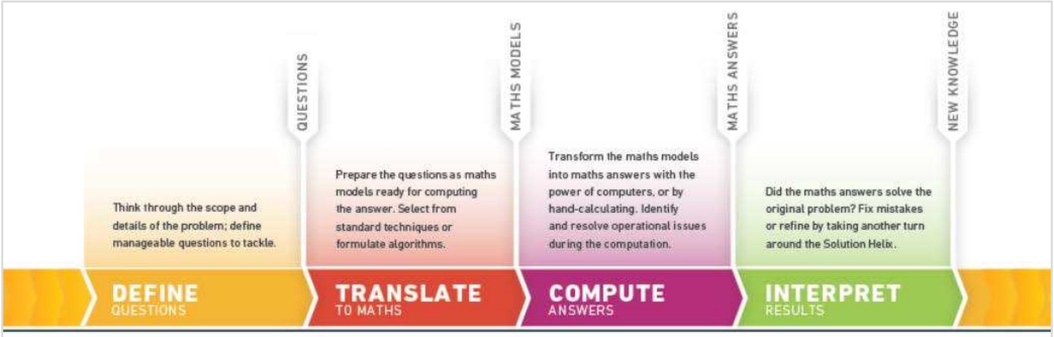
Computer based mathematics in the Applied Mathematics specification

The rationale for Applied Mathematics includes developing problem-solving skills with the result that learners will be able to:

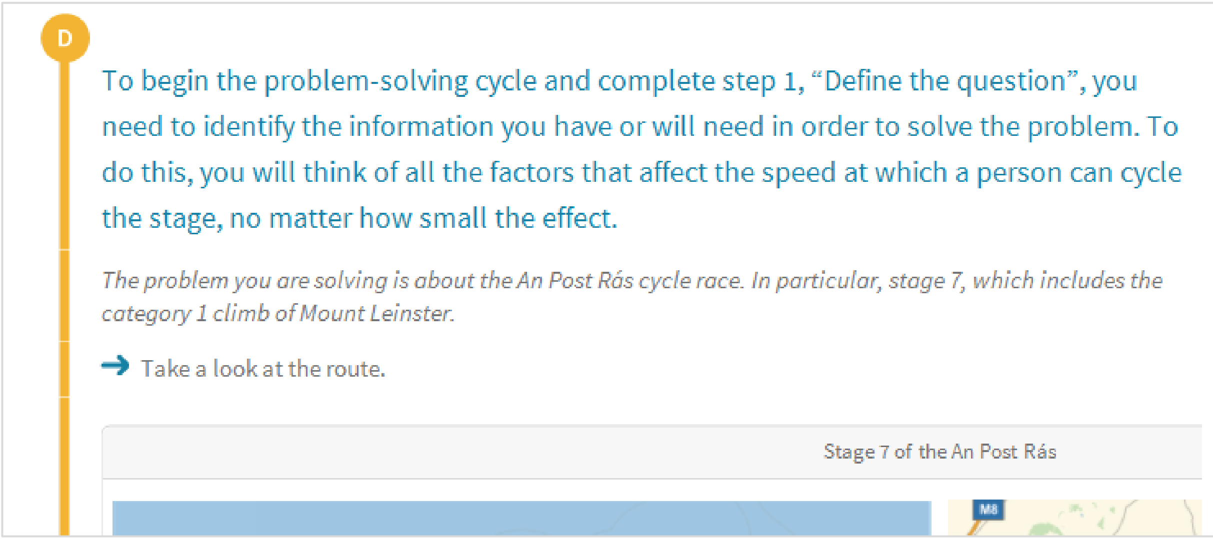
formulate a problem − to consider the scope and detail of a real-world problem, and to define manageable questions to address;
translate problems into mathematics − create or choose a suitable mathematical model, and then to formulate the question as a mathematical problem within the model;
compute a solution − To use mathematical techniques to solve the mathematical problem;
evaluate the solution − To interpret the mathematical solution in the original context.

To reflect this rationale, computational thinking was considered as an important component of the learning, as it is a problem-solving process that includes a number of characteristics and dispositions. Computer-based mathematics is essentially a computational thinking curriculum. It is a way of thinking about life in which you creatively and cleverly apply a 4-step problem-solving process to ideas, challenges, and opportunities you encounter to make progress with them.
In 2015 the National Council for Curriculum and Assessment (NCCA) commissioned Wolfram Mathematics to develop a computer based module for the new Applied Mathemaitcs specification. This module was designed as a proof of concept that exemplified ways in which

students could employ a four step problem solving cycle to solve modelling problems based on authentic contexts. The module is based on a cycle race around Ireland. In solving problems related to this race, students make assumptions, choose a mathematical approach, get a solution, assess the solution for usefulness and accuracy, and then rework and adjust the model as needed until it provides an accurate and predictive enough understanding of the situation. Communicating the model and its implications clearly can be as critical to a model's success as the solution itself. An illustration of the 4 step process is provided below.



Students start by defining the question that they really want to address,



computational thinking follows this with a crucial transitional step; Step2 Translate to mathematics where they take these questions and translate into abstract computational language—that could be code, diagrams, formulae.

> T
>
> To start to form a model that can be used by a computer, we will rewrite the equation for power into a form that can be used by the computer, turning a word equation into code. This is part of step 2, "Translate to maths".
>
> → Read the following process for converting a word equation into code.
>
> *Here's the basic equation that we will be using.*
>
> | Power equation |
> | --- |
> | power = force × velocity |

This has several purposes. It means that 100s of years' worth of figured out concepts and tools can be brought to bear on the question (usually by computer), because they've turned the question into a form ready for this high powered machine to do its work. Another purpose of step 2 is in forcing a more precise definition of the question. This abstraction step is the most demanding of high conceptual understanding, creativity, experience and insight.

After abstraction comes the computation itself—step 3—where the question is transformed into an abstract answer—in this case by a computer.

> C
>
> Once you have a function defined, you need to know how to use it to do calculations and solve equations. In this way you can build a model and solve problems. This forms step 3 of the problem-solving cycle, "Compute".
>
> *How do you know if the computer knows what you have defined for power?*
> *Evaluating* **?power** *will tell you the definition of power. This should NOT give* Symbol power not found. *If it does, evaluate your power function above again.*
>
> → Check you have evaluated your power function by evaluating this code:
>
> ```
> ? power
> ```

In step 4 we take this abstract answer, interpret the results, re-contextualising them in the scope of our original questions and sceptically verifying them.

> I
>
> You have a solution, but is it any good? Here you will interpret your result, whilst verifying and critiquing your model.
>
> *Use your value of velocity to check your model is working correctly.*
>
> → Substitute your velocity and your other values into this calculation to check the power agrees with that predicted by the model.

The process rarely stops at that point because it can be applied repeatedly with output informing the next input until you deem the answers sufficiently good.

Real world mathematics relies on computational thinking

A key difference too between a traditional mathematical approach to a problem and a modern computational thinking approach has to do with the cost-benefit analysis between the 4 steps. Before modern computers, step 3—computation—was very expensive because it had to be done by hand. Therefore, in real life you would try very hard to minimise the amount of computation at the expense of much more upfront deliberation in steps 1 (defining the question) and 2 (abstracting), a very deliberate process. With a computational thinking approach, it is reasonable to expect students to have a much more scientific or experimental approach with a looser initial question for step 1 (like *can I find something interesting in this data)*, an abstraction in step 2 to a multiplicity of computations (like *let me try plotting correlation of all the pairs of data*) because computation of step 3 is so cheap and effective students can try it multiple times without worrying about wastage at that step. Modern technology has dramatically shifted the effective process by avoiding the problem of getting stuck at stage 3.

Conclusiuon

Introducing real mathematical modelling helps shift the perspective on what qualifies as an engaging "real-world problem." As modellers, students will have the opportunity to tackle problems that matter to them and to society. They will decide what information is relevant, make reasonable approximations, use appropriate mathematical tools wisely, and communicate clear, compelling results. As modelling teams, students learn to persevere through challenges and may even surprise us all with the ways they can use mathematics to improve the world. Moreover, the uncovering of the societal role and function of mathematical modelling in the teaching of mathematics can create an important motivation for learning mathematics.
Learnings from the *Project Maths* initiative have provided very useful insights for the developers of the new LC Applied Mathematics curriculum. In particular, it is clear that change in a specification without a change of mind-set is insufficient to bring about the level of change necessary for students to develop the higher order mathematical skills that have been identified as lacking in Irish students. Any new mathematics curriculum should be designed to emphasise the process of mathematics where students create, use, and interpret models of real world situations using mathematical reasoning. Generating rather than recalling mathematical procedure is a critical behaviour that will promote the acquisition of metacognitive knowledge as well as knowledge in the factual, procedural, and conceptual domains. The design of the specification should focus is on learning as an enabling process that helps learners acquire knowledge as they develop capabilities and attributes. Carefully constructed learning outcomes exert a pull rather than a push on the teaching and learning process (Tunstall & Maxwell, 2001) (Maxwell, 2002), (Fensham, 2002), and facilitate the concurrent planning for teaching learning and assessment.
The use of mathematical modelling in society contributes to establishing mathematics as a language that helps students to make sense of the world around them. The inclusion of learning outcomes in the new Applied Mathematics specification, that engage students in critical discussions of everyday data-sets have the potential to help students to navigate between data and the social aspects that underpin them. Across the world, countries are facing common scientific, socio-scientific, economic and political issues that need to be understood systemically in order to be able to make informed decisions about them.

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Grgurina, N. (2016) *Modeling as a New Literacy.* [Working paper submitted for CIDREE-STEM 2016, December 22nd].

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Modeling as a New Literacy

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Abstract

This paper argues that modeling and simulation, being an integral part of computer science (CS), can contribute to the learning and understanding of phenomena in other (STEM)[1] disciplines. Three simulation modeling techniques are discussed and one of them - agent based modeling (ABM) - is found to be suitable for use in schools where students' knowledge of mathematics and phenomena they study is limited.

Modeling in Science

The history of science shows it evolved through several paradigms: from the empirical science focused on describing natural phenomena in ancient times, through theoretical science looking for generalizations and developing models in recent centuries, to computational science that simulates complex phenomena since the arrival of computers, and further to data science seeking to unify theories, to experiment and run to simulation [8]. Obviously, modeling plays a significant role in the development of scientific knowledge.

Models should also play a significant role in science education where the purpose of learning science is threefold:

- to learn science;
- to learn about science; and
- to learn how to do science.

To help achieve these goals, students should learn about the existing models, the nature of models, and they "should be able to create, express and test their own models" [9]. Through the availability of modeling tools and techniques, CS can contribute to achieving these goals.

Modeling in Schools

A number of initiatives have already been employed to aid students' learning in various (STEM) disciplines through the use of computer models for e.g. physics [10; 11], material science [3] and biology [1]. Nowack and Caspersen [5] argue why they „believe understanding and creating models are fundamental skills for all pupils as it can be characterized as the skill that enable us to analyze and understand phenomena as well as design and construct artifacts." Wilensky claims, „Computational modeling has the potential to give students means of expressing and testing explanations of phenomena both in the natural and social worlds" [12]. Granger goes even further and declares, „Modeling is the new literacy" [7]. While that statement can be a subject to debate, it is certain that modeling, together with simulations, can be a valuable part of a (STEM) curriculum. This fact is recognized in the new Dutch curriculum for the elective CS course in the grades 10 through 12 of the senior general secondary education (Dutch: HAVO) and pre-university education (Dutch: VWO) by including modeling in the core curriculum and computational science (i.e. modeling and simulations) in the elective part of the curriculum[2]. The rationale for the inclusion of these learning objectives into the curriculum is guided by the fact that many students electing the CS course are going to pursue scientific and engineering careers outside of CS, and CS can provide them with the skills that allow them to formulate problems in such a way that these can be solved with the help of a computer. In other words, this is the recognition of the fact that CS can provide methods and ways of thinking that can contribute to the understanding of phenomena in other (STEM) disciplines.

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3. Agent Based Modeling

The STEM disciplines often employ scientific modeling, „the generation of a physical, conceptual, or mathematical representation of a real phenomenon that is difficult to observe directly" [6]. Building physical models in a (STEM) classroom has its merits but generally does not happen in a CS class, and finding a non-trivial mathematical representation of an arbitrary complex phenomenon is often beyond the reach of K-12 students. Conceptual representation, however, is well within reach of these students when appropriate techniques and tools are used. Conceptual representation with the aid of computing, where a model has a form of a computer program and is used to run simulations, is simulation modeling. There are three methods in simulation modeling:

- System dynamics, associated with high level of abstraction where the individual objects are aggregated. The models are described in terms of coupled nonlinear, first-order differential equation. Solving these equations is often non-trivial and requires the use of numerical methods[4]. High demands on mathematical knowledge deem this type of modeling out of reach for most secondary CS students – an assertion supported by observation in my own classroom and corroborated by Wilensky [12].
- Discrete event modeling, where the system modeled is considered to be a process, „i.e. a sequence of operations being performed across entities". The level of abstraction is lower as „each object in the system is represented by an entity or a resource unit" that are passive, i.e. the process flowchart defines what happens to them [4].
- Agent based modeling (ABM), which is made possible with recent growth of availability of CPU power and memory. It does not assume any particular abstraction level. Agents have their properties and behavior and one can start building a model by identifying agents and describing their behavior even without knowing how a system behaves as a whole. ABM makes it possible to model systems that are difficult to capture with older modeling approaches[4].

The last two characteristics of the ABM make it a suitable modeling method for the students who often lack deep understanding of the phenomena they model, and who make models specifically to deepen their understanding of these phenomena. To conclude, the conceptual representation which could be realized through the employment of ABM methods and software, in which „you give computational rules to individual agents and then observe, explore [and] analyze the resultant aggregate patterns[12]", is suitable for use in a secondary CS class „because the individual-level behavior of agents is relatively simple, [and] ABMs feature relatively simple computer programs that control the behaviors of their computational agents" [13].

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Bech, E.M. (2016) *Pilot project with computer programming in lower secondary school*. [Working paper submitted for CIDREE-STEM 2016, December 22nd].

Pilot project with computer programming in lower secondary school

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Background

Pupils in primary and secondary education in Norway do not receive training in computer programming or informatics as a separate compulsory subject. Computer programming is not a part of other compulsory subjects.

This school year (2016-2017), The Norwegian Directorate for Education and Training has started a national pilot project with computer programming electives in lower secondary schools. The pilot project will last for three years and is a part of the STEM- strategy for 2015-2019 of the Ministry of Education and Science.

The purpose of the pilot

- To gain experience of teaching computer programming in lower secondary schools in order to offer a subject relevant to this peer group in the long term
- To increase students' skills in computer programming
- To increase teachers' skills in computer programming, which is necessary to give students a good education. There are few teachers with formal qualifications in computer programming in elementary school.

The pilot

155 lower secondary schools across the country are participating in the pilot in 55 municipalities and owner of private schools. They must report annually to the Directorate about their experiences with the pilot. The Directorate reports to the Ministry of Education and Research.

<u>Curriculum and guidance in computer programming</u>

The curriculum consists of two main subject areas, modelling and program code.

Modelling

Modelling covers the steps required to solve problems using computer programming, also known as algorithmic thinking. It is about what kinds of problems a computer can solve, how these problems can be broken down into sub-problems and how solutions can be designed. Modelling of mathematics and natural phenomena are central parts of this. How computers and programs are designed and work, various programming languages, and strengths and weaknesses of the various languages are parts of this main subject area. Principles that underlie good programming practices are also included in the main area, including explanation and documentation of solutions and program code.

Program code

Program code is about developing one`s own program code using various programming languages. This includes using and understanding the basic principles of computer programming, such as loops, tests, variables, functions and simple user interaction. It is about control or simulation of physical objects such as robots, sensors, bouncing balls and molecules moving. Furthermore, it relates to simulations and calculations based on mathematical and natural science issues. The main area also includes debugging, generalization and reusing solutions, including assessment and analysis of their own and others' program code.

The guidance

The guide gives advice and tips on how to work with the curriculum. The guidance is not intended to be comprehensive in that examples covering all the competence aims of the curriculum, but it does provide some tips and advice on how teachers can work practically with the academic content of the subject.

The following practical examples are parts of the guide:

- Programming without computer
- Running a code hour with students
- Ball bounce
- From problem to sub problems (workflow)
- March landing
- Robot Vacuum Cleaner
- Play Project
- Pupils resource

Increase teacher competence in computer programming

A part of the pilot is to increase the teacher's knowledge about how to teach computer programming. It is a challenge that students have varying degrees of knowledge and skills in computer programming. How should teachers attend to students who know a lot when the teacher has little formal training in this subject?

To increase teacher' expertise in computer programming, The Directorate and the Norwegian Centre for ICT in Education will organize two conferences a year. In addition, the center for ICT have made a MOOC (massive open online courses) with modular training in computer programming. The center of ICT also offers different courses in several programming languages, both block-based and text-based.
https://kurs.iktsenteret.no/courses

Correlation between Curriculum for computer programming and Curriculum for the common core subject of mathematics for lower secondary school

Informatics is not integrated into The Curriculum for mathematics, but there are goals in both curricula that facilitate that programming can be integrated into mathematics.
Some teachers use computer programming in math lessons. Web based resources with programming can be used in their teaching lessons.
Programming give students the opportunity to work with problem solving, to experiment and make calculations with large numbers. Programming can give students both procedural knowledge and conceptual knowledge in mathematics.

Example of procedural knowledge – problem solving is emphazised in both curricula

Informatics, goals for computer programming

- undo problems to specific sub-problems, assess the sub-problems that can be solved digitally, and design solutions for these. Develop and debug applications that solve defined problems, including science issues and control or simulation of physical objects
- transfer solutions to new problems to generalize and adapt existing application code and algorithms.

Bech, E.M. (2016) *Pilot project with computer programming in lower secondary school*. [Working paper submitted for CIDREE-STEM 2016, December 22nd].

These goals can be related to the following goals for mathematics:

- analyse complex problems, identify fixed and variable quantities, connect complex problems to known solution methods, carry out calculations and present the results in a suitable manner
- use numbers and variables in exploration, experimentation, practical and theoretical problem solving and technology and design projects

Example of conceptual knowledge

In lower secondary school many students lack understanding of basic concepts in mathematics. Computer programming help students understand the basic mathematical concepts through concretization of abstract concepts and experimentation. Algorithmic thinking will help students to develop ability to abstraction and ability to understand mathematical concepts.

These goals for mathematics can be related to informatics

- describe the characteristics of two- and three-dimensional figures and use them investigate and for constructions and calculations explore, experiment with and formulate logical reasoning by means of geometric ideas, and elaborate on geometric relations that are particularly important in technology, art and architecture
- find and discuss probability by experimenting, simulating and calculating in day-to-day contexts and games

Discussions

- There are few teachers with formal qualifications in informatics or computer programming in elementary school in Norway. How is the situation in yours countries? What have you initiated to increase teachers' competence in informatics?
- Do you have informatics as a separate subject or is it included in other subjects? If so what types of subjects?
- How can an increased focus on programming lead to increased learning in other subjects?

Krajnc, R. (2016) *Present status of informatics and its presence/inclusion as an auxiliary tool for learming mathematics in Slovenia*. [Working paper submitted for CIDREE-STEM 2016, December 22nd].
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Present status of informatics and its presence/inclusion as an auxiliary tool for learning mathematics in Slovenia

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Abstract

Informatics is a scientific discipline with its basic knowledge, set of techniques and methods for problem - solving with specific way of thinking and functioning. It is divided into sections like algorithm and complexity, discrete structures, basics of programming, etc. In the Slovenian primary and secondary schools informatics is not included as mandatory for all pupils, while on the other hand in the high schools there is a subject called informatics, which is mandatory for one year, but within this year mainly digital competencies and the use of digital technology is taught.

The use of ICT (Information Communication Technology) is a part of didactic recommendations in the curriculum, and ICT means the use of digital technology in learning and teaching process. The amount and meaningfulness of ICT usage varies among different subjects.

Mathematics is one of the basic subjects in primary school and it develops mathematical competence. By that we mean the ability to use mathematics way of thinking for solving mathematical problems and real-life problems. Within mathematical competence also the use of ICT is developed.

ICT is used in the process of acquisition of new terms, concepts, procedures and in inquiry based learning, problem solving in mathematical and authentic context. We note that informatics curriculum is not part of the mathematics curriculum. The same situation is in other STEM subjects.

Introduction

We shall describe more specifically the inclusion of informatics, digital literacy and the use of ICT in the Slovenian curriculum. As it is necessary to know the history of the subject´s development in order to describe the informatics present status in the Slovenian school system, certain details are quoted, which will help us shed more light on the present status.
In Slovenia, informatics appeared for the first time in a secondary school back in 1971. At that time informatics was taught at several secondary schools by the informatics experts, while simultaneously the future teachers of informatics were also being trained. In the first year, the informatics classes were attended by 200 pupils, while in the following years their number was steadily growing. In 1974 the informatics classes were attended by 2500 pupils. The classes were taught primarily in a theoretical way, while the emphasis fell on the algorithms and programming languages; thus the Fortran language was primarily taught. In 1980 secondary schools of informatics and computer programming were founded, The subject called informatics started to be introduced in the primary schools in 1985. At that time the teachers, who primarily carried out the extra-curricular activities, started to be trained in informatics. In 1988 and 1989 in the projects Raček and Petra the schools started to receive the hardware and software, while the teachers could take part in the education and training of using the new technology during classes. In 1990 the informatics became the mandatory subject at four High schools of informatics in the first year. As the subject was not placed among the optional matura/final exam subject, the interest for it in senior classes started to wane and consequently, informatics in

senior classes was taught less and less. In 1991 and 1994 the level of informatisation in Slovenian schools started to systematically increase. In five years more than 31000 teachers were included in the informatics training and education. In new projects (1999/2000) the training and education in ICT for the use during classes continued, while the schools were being equipped with the adequate computer equipment. Unfortunately the term informatics became synonymous more and more with the use of ICT, and consequently during the subjects, named informatics, the use of tools, as MS Word and Excel, as well as video editors etc. were taught.

Present status of informatics in the primary school

With the introduction of the nine-year long educational period into the primary school, the pupils had a chance to select an optional subject - informatics. The curriculum regarding the compulsory optional subject informatics was created in 1998 and has remained unchanged until today. The subject can be chosen by the pupils in the last three years of education in the primary school (the pupils´ age: 12-15 years) . Beside getting to know the basic terms and laws, the pupils also indirectly work with computers, thus getting the useful knowledge regarding the use of ICT. The curriculum predicts in the first year the work with the word processors, in the second year they get to know the computer networks, in the third year multimedia. The subject is practically-oriented, with pupils developing digital literacy. The chapter on programming as optional is also included in the curriculum, but it is not compulsory. As the subject itself is optional, it is up to the pupils themselves, whether they choose to attend the subject or not. The teachers thus report that the number of pupils, who select informatics, is falling. In certain schools the subject itself is not carried out any more due to the lack of interest among pupils.
In 2014 new non-compulsory optional subjects started to be carried out in Slovenian primary schools. One of the five optional subjects that can be chosen by the pupils in the 4th,5th and 6th class (pupils´ age: 9-11years) is Informatics. The curriculum for this new subject was confirmed in 2013. The subject is entirely intended for the creation of the computational thinking, problem-solving, algorithm familiarisation and programming. The curriculum is made up of five blocks (Ministry of Education, Science and Sport,2013,pp 5-7): algorithms, programmes, data, problem-solving, and communication and services. Pupils get to know the tools for visual programming Scratch (Lifelong Kindergarten Group, MIT Media Lab, 2016), also getting to know the sequence of command performance and multi - string sequence, loops, branch programming, variables, responding to events, input data reading and similar.
As the subject is non-compulsory and optional, only 10% of all the pupils of the 4th class in Slovenia chose it. The pupils during the subject lesson usually use the tool Scratch, while those pupils, having attended the subject lessons for one or two years, can use in their third year for their own projects TouchDevelop. Teachers of this subject are faced with various challenges, as they can have in a group mixed pupils from 4th,5th or 6th class, while the same group can also unite those pupils, having attended the same subject classes, thus possessing certain knowledge, and those without any experiences in this field. Beside a group can have up to 28 pupils, while the computer classrooms have only 16 PCs. In spite of this the new subject can be for at least some pupils an opportunity to get to know the basics of informatics.
The pupils can also see the computer contents during computer sciences and programming extra-curricular activities, as well as other activities. They can also take part in various computer sciences and programming competitions.
To sum up: in the Slovenian primary school within the compulsory curriculum, there are no informatics contents. The pupils have an opportunity to get to know the informatics contents only in the 4th,5th and 6th class in the extended part of the curriculum (under optional contents), meaning that only a small percentage of population meet the informatics in the primary school. The use of the information communication equipment is also included in the currently valid curriculum of most subjects under the didactic recommendations. These recommendations regard the use of certain tools, which belongs to the field of information technology and not

informatics. In the following text, the current status is presented in regard to the subject of mathematics.

The curriculums also include development of digital competences. The European commission workgroup in 2013 published the DigComp framework, outlining the subjects of focus and digital competences. Each one of 21 digital competences is thus described to a greated detail in the model, which can be helpful for teachers when planning the lecture. Digital competences must be systematically and specifically developed throughout the entire school years in all subjects. The school practise shows that schools meet different challenges and problems when developing digital competences. In spite of various projects and a great number of educational courses, the teachers do not develop their digital competences systematically and specifically by agreement with other colleagues. The reason may lie in the simple fact, namely in the overabundance of contents in curriculums and the lack of time for the digital competences development.The computer classrooms and other smart mobile devices are not that available, reducing the teachers´ interest for the ICT use. All schools also do not have a required infrastructure, enabling a rapid, safe and simple use of the networks for the pupils, who might bring their own device to the school (BYOD).
One of 21 digital competences within DigComp is also programming. This is competency 3.4 regarding the creative contents. It is therefore possible that a teacher of any subject could in principle for achieving their goals include programming, but in order to do that, he/she should be appropriately trained and qualified, which in reality the teachers of non-informatics subjects are in most cases not. In order to make such classes successful, the pupils would also have to have certain informatics know-how, but unfortunately the pupils in Slovenia within the compulsory syllabus do not acquire it, and consequently programming is usually not included in the classes.

Present status in grammar schools

In grammar schools (gimnazija) informatics is an obligatory subject only in the first year. The students can otherwise also select informatics for the final exam - in this case a student can attend informatics classes every year for 2 hours per week.

Regarding the subject, it is intended for the development of digital competences. Regarding a more detailed description of goals and contents, the subject scope also encompasses the informatics matter (Hardware,Software,Networks, Programming), but due to the curriculum´s vagueness, the teacher has the right to choose the informatics contents in the mandatory first year. In most cases the first year students meet the basic informatics terms, namely, word processing, spreadsheet editing and solving an information problem. The main reason that the teachers do not include more informatics contents in the first year is in the fact that students come from different primary schools with different knowledge and in most cases with poor digital literacy. Due to this most teachers are of the opinion that the students must be first made digitally literate, and only after this process they can get to work with the informatics contents. The situation in this field is slowly improving, as certain teachers have lately included programming into the mandatory first year subject informatics.

Mathematics in the Slovenian school curriculum

Mathematics is an obligatory general educational subject in the primary school and in all programs of the secondary education in Slovenia. The number of mathematics lessons regarding individual classes of the primary educational program is shown in the Table 1, while the number of lessons in individual secondary school program is shown in the Table 2.

Krajnc, R. (2016) *Present status of informatics and its presence/inclusion as an auxiliary tool for learming mathematics in Slovenia*. [Working paper submitted for CIDREE-STEM 2016, December 22nd].

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Table 1. *The number of math lessons in the compulsory primary school*

The compulsory primary school program	***Prescribed number of math lessons per week***
1. class	4
2. class	4
3. class	5
4. class	5
5. class	4
6. class	4
7. class	4
8. class	4
9. class	4
Total	1318

Table 2. *Number of math lessons in different secondary school program*

Secondary school program	***Number of years of the program duration***	***Total math lessons for the program[52]duration***
Upper secondary general education (gimnazija)	4	560
Upper secondary technical education	4	383-408
Upper secondary vocational education	3	206-242
Vocational-technical education	2	213
Short upper vocational secondary education	2	157

The math lessons goals

The math teaching is in its most general term intended for building the terms and connections among themselves, getting to know and learning the procedures, enabling an individual inclusion into the system of (mathematical) ideas and consequently inclusion into the culture in which we live (Žakelj, 2015).

In the General plan for the teaching of mathematics for the primary school (UN, 2011) it is defined what the general goals of mathematics are: the pupils during the math lessons:

- develop mathematical thinking: abstract-logical thinking and geometrical concepts;
- form mathematical terms, structures, skills and processes, connecting their knowledge within mathematics and also more generally;
- develop the use of different mathematical procedures and technologies;
- get to know the applicability of mathematics in everyday life;
- get to know mathematics as a process, while also learning creativity and precision;
- develop trust for their own (mathematical) abilities and positive relationship to work and mathematics;
- get to know the meaning of math as a universal language;
- accept and experience mathematics as a cultural value.

The goals of the mathematics teaching in the grammar school represent the continuation and upgrading of goals from the primary school. The students should learn:

- develop mathematical thinking: abstract-logical thinking and geometrical concepts;

[52] The number of maths lessons in individual year classes is not prescribed with the curriculum on the state level, but the decision regarding the distribution of them is solely left to the individual school.

- learn about the construction of the mathematical theories and basic standards of mathematical conclusion;
- get to know the questions, on which math can offer answers to ;
- get to know the meaning of math as a universal language and tool;
- express oneself in a mathematical language in an oral, written or other expressive forms;
- use math in contexts, connecting the knowledge within math and also more generally (cross-curricular integration);
- ask key questions, emanating from the life situations or connected to the research of mathematical problems;
- get to know math as a process, develop creativity and to trust in one´s own mathematical abilities;
- **get to know and use different information-communication technologies (ICT) as a means of help for a more effective learning and problem-solving;**
- **assess when it is meaningful to use a certain information-communication technology and develop the critical thinking towards information on the web**.

Among many math lessons´ goals in the secondary educational programs the following are emphasized:

- Understanding and ability to use the basic mathematical terms and relations among them, as well as the procedure performance
- Ability to research and solve problems in mathematics
- Ability for generalisation and abstraction and problem-solving on general or abstract level
- Ability for interpretation and critical judgment when using mathematics in professional and other areas
- Ability to use the mathematical tools in communication
- **Ability to use the technology in performance of mathematical procedures and in the research and solving of the mathematical problems**
- Ability to collect, organise and analyse the data
- Ability to plan and organise the operation
- Ability for cooperation and team work
- Responsibility for the personal knowledge and ability for the independent learning of the mathematical knowledge
- Acceptance and experiencing math as a cultural value
- Trust in one´s own mathematical abilities in creation of the positive self-worth

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The role of technology in math lessons

In achieving the math lessons´ goals, the information-communication technology, which in general is increasing its influence more and more in the modern world, has a very important role. It can be found in different existing forms in all areas of human activity from economy, health, to media, entertainment industry and human personal life.

In the Catalogue of knowledge for the secondary professional education one can thus read that » all the professional fields are getting more and more mathematicised, while math itself is less and less visible in them, hidden in **technology (computer programs, mathematical models, machines and products)**. In order to perform certain activities, the ability of calculation and certain mathematical procedures are less and less important, whereas understanding of mathematical terms, the ability to connect the mathematical knowledge with the given situations in a certain professional field and ability for solving the professional problems are on the other hand more and more important. This is the basis for a competent use of the technological tools available today, doing the major part of the basic mathematical procedures. «

Similar information can also be found in the Curriculum for the primary school and in the Curriculum for grammar school, both stating that »with the development of the information-communication society the presence of math in other subject fields is less and less visible, as it is hidden in **technology.** In order to run certain activities, the routine mastery of calculation procedures is therefore less important, while more important being understanding, cross-curricular integration and the use of mathematical knowledge and the ability to solve problems «.

Didactic recommendations for teachers on which technology they have at their disposal, in what way to use it during math lessons and what its role should be follow the records in curriculum plans.

There are various types of technology available:

- numerical and graphing calculators,
- symbolic calculators,
- personal or portable computer with computer programs (dynamic geometry, function programs, computer spread sheets, statistics programs, programs for learning or revising certain mathematical contents, programs intended for the development of mathematical terms, programs intended for the automatization of knowledge and its examination…)
- internet (information, electronic learning material, e-mail, internet classrooms, video conferences …),
- tools and programs for data or operation results recording and presentation (interactive board, presentation programs…).

The teacher selects the most suitable tool regarding the goals and standards (or expected achievements), the method of work, the approach to the subject treatment, population´s characteristics and personal affinity. The sensible and critical use of ICT in schools, based on the realization that the math lesson has a higher added value than the math lesson without the use of ICT, is encouraged. At the very same time, the active use of ICT, where the central role is on the pupil him/herself, who with his/her mental activity with the support of ICT builds up his/her math knowledge, is also encouraged.

The role of ICT regarding the math lessons encompasses:

- development of mathematical terms,
- means for creating, research, simulation and modelling of the real and teaching situations,
- merely a learning tool,
- work method,
- communication device,
- means for following and checking on knowledge.

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Even in case that ICT during the math lesson is used only in a function of demonstration, its advantages are made good use of (Kmetič in Suban, 2013):

- potential saving in time and the option for treatment of exercises of higher taxonomic levels (e.g. tabulating functions, drawing of the function graph, calculation of a higher number of similar calculations),
- clearness, transparence, visualization,
- picture dynamics (e.g. dynamic geometry programs),
- possibility of multiple repetitions of experiment (´what if´ analysis),
- option of getting a fast, objective feedback for a great number of students simultaneously.

It could be said that such a role of ICT in math lessons does not guarantee systematic and planned development in the knowledge of informatics and computer sciences, as the knowledge of programming and algorithms is excluded. The qualification of math teachers in order to prepare the activities, during which the students would be solving the math problems by using their knowledge of informatics, is also questionable.

Conclusion

Regarding the things explained above, we think that the whole field of informatics should be properly sorted out and vertically included in the whole educational system. With the possible future change of the curriculums, informatics should also be included into other subjects, as for example mathematics, physics, chemistry, biology, language.

Ministry for Education,science and sport has this year founded a working group of experts, tasked to analyze the status of informatics in Slovenia, as well as abroad and to prepare a plan and proposition regarding the introduction of informatics into the school system. The deadline is December 2017. The proposition will then be checked by the education minister, and will then put it into a public debate at his/her own discretion. The path to introducing informatics into schools is still very long, as the possible changes will have to be ratified in the parliament. Before that the experts should also unify regarding the said changes and modernisations, requiring a serious time for thinking and a wide public debate. We are unfortunately faced with the maintenance of the present state, in which the students do not receive the required knowledge, making them competitive on the global market of the work force.

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Karis, H. & Andersson, O. (2016) *Proposed curricula and in-service-training concerning digital competence.* [Working paper submitted for CIDREE-STEM 2016, December 22nd].
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Proposed curricula and in-service-training concerning digital competence

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Introduction

This paper, together with our seminar in Utrecht, presents the Swedish on-going curriculum work concerning digital competence. It also includes a presentation of an in-service training of teachers, since the Swedish National Agency for Education (hereinafter abbreviated as the Agency) have a mission to implement the revised curricula and to support teachers in developing computer based education.

The aim of the ongoing curriculum work is to highlight digital competence and the use of digital tools, sources and media for more effective learning and higher learning outcome in the curricula. Digital competence includes programming as a tool or method for problem solving and control and regulation technology.

This paper and the presentation starts with an overview of the structure of Swedish steering documents and how the digital competence is explained and expressed in the Swedish curriculum and course syllabuses, especially in mathematics, technology and science. It is followed by a description of how the Agency support teachers developing computer based education, which includes digital competence and how to use digital tools and media for a higher learning outcome and a more effective learning.
In the end of this paper there are some questions for discussion about programming as a tool or method for problem solving and control and regulation technology.

The Swedish steering system

The Swedish school system is a goal based system with a high degree of local responsibility. It is not possible to include pedagogical guidelines in the Swedish curriculum and course syllabuses. *How to teach* is considered to be the teacher's domain. All guidelines and principles are considered as intrusions on teacher's professionalism.

The main responsibility for educational activities lies with the municipalities and the local authorities responsible for independent schools. The steering documents are:

- The Education Act, (Skollagen)
- Curriculum for the Preschool, (Lpfö 98)
- Curriculum and knowledge requirements for the compulsory school, preschool class and the recreation centre, (Lgr 11)
- Curriculum for the Non-compulsory School System, (Lgy 11)
- National tests for grade 3, 6, 9 (comp school) and for each course in upper secondary education (non-comp school)

Improving digital competence

The Agency was in the year of 2015 given a government mission to suggest enhancements of all curricula in the Swedish school system regarding digital competence. The changes will most likely be in full use in August 2017. The government mission had focus on specifying goals through revising the curriculum, course syllabuses and on improving quality of teaching and support teachers in the classroom.

Karis, H. & Andersson, O. (2016) *Proposed curricula and in-service-training concerning digital competence.* [Working paper submitted for CIDREE-STEM 2016, December 22nd].

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Digital competence in the curriculum

The ongoing work with curricula in the Swedish school system, with the purpose to enhance and specify digital competence, had to start with a definition or description of digital competence. OECD's definition of digital competence in 21 century skills was used in the work. The Agency described digital competence when working on the steering documents like this:

> *Digital competence is made up of the extent to which you are familiar with digital tools and services and have the ability to keep up with the digital revolution and its impact on your life.*
>
> *Digital competence includes:*
> - *skills to search information, communicate, interact and produce in a digital way*
> - *skills to use digital tools and services*
> - *understanding of the transformation that digitization means for society, with its opportunities and risks*
> - *motivation to participate in the digital development*

In the curricula for compulsory school, digital competence is worded like this:

> *The school should stimulate students' creativity, curiosity and self-confidence as well as their willingness to try and put their own ideas into action and to solve problems. Students should have the opportunity to take the initiative and responsibility, and develop their ability to work both independently and together with others. The school will help students develop an understanding of how digitization affects the development of society and us as individuals. Students should be given the opportunity to develop their ability to use digital technology. They should also be given the opportunity to develop a critical and responsible approach to digital technology, in order to see the possibilities and understand the risks and be able to evaluate information. The school should thus give students opportunities to develop digital skills and an approach that promotes entrepreneurship.*

The process

The process of revising all curricula regarding digital competence started in October 2015 by forming a project group consisting of directors of education from the Agency. The group started the work by going through all curricula and making an inventory of content connected to digital competence to be able to identify parts that needed to be revised.

The Agency also wanted to know if teachers already worked with digital competence and programming in their school subjects and published a survey on social media (Twitter and Facebook). About 400 teachers answered the survey and the result was that most teachers worked with programming in mathematics and technology.

The Agency initiated two research summaries on programming in schools in Sweden and other countries and recent changes in other countries curricula regarding programming. After identifying what subjects to focus on in the revision process the Agency started subject oriented reference groups consisting of researchers in computer science and didactics, teacher trainers and teachers. These groups discussed changes in the curriculum and course syllabuses and later in the process drafts were sent to the groups to comment on. At the same time the Agency participated in several workshops all around Sweden to talk about the work with the curriculum and also arranged meetings with teacher unions, student's organizations and other governmental educational agencies in Sweden.

In the spring of 2016 drafts had reached the quality level to be published at the Agency's web for anyone to comment on. The Agency worked through all the comments and in May, all curricula and course syllabuses were sent on a formal referral to authorities, municipalities and organizations. In June the report was sent to the government and the Agency are now waiting for their response.

All the external contacts during this process are seen as part of the future implementation of a revised curriculum.

Curriculum changes concerning digital competence in mathematics, technology and science

Below is an overview of the syllabuses in mathematics, science and technology in the suggested curricula.

a) Mathematics

Extract from the aim of mathematics:

> *Through teaching, pupils should be given the preconditions to develop their familiarity with basic mathematical concepts and methods, and their usefulness. In addition, through teaching pupils should be given opportunities to develop knowledge in using digital technology and programming to explore problems and mathematical concepts, make calculations and to present and interpret data.*

Extract from mathematics core content years 1-3 (7-9 years old)

- *How clear step by step instructions can be constructed, described and followed as the basis for programming. Using symbols in step by step instructions.*

Extract from mathematics core content years 4-6 (10-12 years old)

- *How algorithms can be created and used in programming. Programming in visual programming environments.*

Extract from mathematics core content years 7-9 (13-15 years old)

- *How algorithms can be created and used in programming. Programming in different programming environments.*
- *How algorithms can be created, tested and improved in programming for mathematical problem solving.*

b) Science

In the core content in biology, chemistry and physics years 1-9 (7-15 years old) the Agency have added text about using digital tools for documenting purposes, using digital media as a source of information and using simulations to support modeling in science. In physics there are also added core content on the use of electrical sensors for measurement.

c) Technology

Extract from technology core content years 1-3 (7-9 years old)

- *What computers are used for and some of the computer's basic parts for input, output and storage of information, such as keys, screen and hard drive. Some common objects that are controlled by computers.*

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- *To control objects with programming.*

Extract from technology core content years 4-6 (10-12 years old)

- *Some of the computer's parts and their functions, such as CPU and RAM. How computers are controlled by software and can be a part of a network.*
- *To control own designs or other objects with programming.*

Extract from technology core content years 7-9 (13-15 years old)

- *Technical solutions for information and communication technology for the exchange of information, such as computer, internet and cellphone.*
- *Technical solutions using electronic equipment and how they can be programmed.*

Implementing Curriculum

The process of implementing a revised curriculum consists of different parts. The process is similar for all changes made by the government. Teachers, principals and local school authorities need *information* about the changes as well as *support for implementing* the curricula changes in the local school. There could also be a need of *more knowledge* in the subject, for example to learn more about programming.

When it comes to education material and teaching methods the Swedish school system considers the “How to teach”- question to be the teacher’s domain. The teacher is responsible, together with the principal, for developing teaching. In general the government is not producing education materials. There is no text book reviewing made by the government either.

In Sweden, the Agency is not responsible for higher education (e.g. teacher training at university level), but it is likely to assume that they play an important role in the implementation of future changes in the curriculum.

Implementing digital competence

When it comes to implementing the changes in the curriculum it is important to reach the entire school system including local school authorities, principals and teachers. The Agency supports the local authorities, principals and teachers in purpose to improve the digital competence through in-service training in programs. The Agency is now developing programs for both principals and teachers. There is one program about digital competence for principals called “Steering and managing digitalization in schools” and there are a number of programs for teachers: insatser

- Safe use of resources on Internet
- Digital tools for learning
- Leading learning in technology rich classrooms
- Digital tools for including students with special needs
- Digital competence and programming
- Remote teaching

In-service training supported by the Agency

Later years, since 2012, in-service teacher training has been increasing as mission from the government. Today it includes hands-on didactics in all subjects, but with focus on reading, writing, mathematics and science. It also includes targeted initiatives to support schools with low outcome, “Tutorial for learning”.

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Digital competence is focus the next year for the in-service-training and all programs above are designed basically in the same way as earlier programs launched by the Agency, for example *The boost of mathematics* (Matematiklyftet, https://matematiklyftet.skolverket.se) *Science and tehnology program* (NT-satsningen www.skolverket.se/nt).

An important part of the programs is peer learning. In- service training and support of different levels in the education system are effective for higher results. Researchers, for example Hattie tells us:

- Nothing is more effective than a skilled teacher
- Peer learning among teachers, with external support is effective for higher results
- An active, engaged, participating and driving school leadership is a prerequisite for a school development program to be successful

Implementing programming in mathematics and technology

One big difference this time is that we think that many teachers in mathematics and technology lack prior knowledge in programming so the teachers will need some basic training in programming before they can take on in-service peer learning programs.

Evaluation

Since there's not that much research made about digital competence and programming in schools in Sweden, it is important to closely monitor and evaluate the results of the changes in the curriculum. The Agency suggested the government to reserve money to support research in this field. The Agency also needs to be prepared for quick revisions of the curriculum based on future evaluations and research on the changes in the suggested curriculum.

Discussion

Below there are some questions that can be discussed at the CIDREE meeting.

a) Programming knowledge

- What prior knowledge is essential when you start programming?
- What knowledge in mathematics is essential for being a good programmer?
- What learning resources are the best and most effective for learning programming? What is the learning outcome?
- Some teachers tell us that programming takes time from and excludes mathematics. What is your opinion?

b) Programming/coding as a tool for Problem-solving and Control and Regulation Technology.

- What is the relation between mathematics and computer science?
- What learning resources are the best and most effective?
- What is the learning outcome? What research do we have regarding this?

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